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Abstract

Two sided markets involve two groups of agents who interact via "platforms". This paper analyses competition in a two sided market with congestion. The existing literature's on pricing mechanisms of two-sided markets has concluded that pricing mechanism depends on the following three factors: relative size of cross group externalities, fixed price or per transaction charge by platform, and single homing or multiple homing of agents. This paper extends the analysis by including the effect of congestion on pricing mechanisms in a two sided market. It concludes that in the case of single homing of agents, profits of the platform increase due to congestion if the agents have a low tolerance level, whereas in the case of multi homing, profits of the platform increase due to congestion if the agents have a high tolerance level.

Keywords: Network externalities, Congestion

JEL Code: L10, L11, L14, D43

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Two sided markets involve two groups of agents who interact via "platforms". This paper analyses competition in a two sided market with congestion. The existing literature's on pricing mechanisms of two-sided markets has concluded that pricing mechanism depends on the following three factors: relative size of cross group externalities, fixed price or per transaction charge by platform, and single homing or multiple homing of agents. This paper extends the analysis by including the effect of congestion on pricing mechanisms in a two sided market. It concludes that in the case of single homing of agents, profits of the platform increase due to congestion if the agents have a low tolerance level, whereas in the case of multi homing, profits of the platform increase due to congestion if the agents have a high tolerance level.

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1 Introduction

There are many markets in which two different sets of agents interact with the help of platforms. One set's benefit in joining the platform depends on the number of agents in the other set who join the platform. This type of market is known as a "Two Sided Market". Some famous examples of two sided markets are: cab aggregators like UBER and OLA (where riders would want to use an app on which they can get a cab easily i.e. a platform on which there are many drivers and drivers would want to join a platform that has many riders), universities (students would want to join a university that has good teachers whereas a teacher would want to join the university that has a large number of good students), television channels (viewers typically prefer to watch a channel which has less commercial ads whereas an advertiser is willing to pay more to place the ad on a channel with more viewers), credit cards, night clubs, heterosexual dating apps and shopping malls.

In the last two decades, the reach of internet has increased manifold and so have the examples and the literature on the two sided markets. Some of the pioneering works on two sided markets have developed group-specific pricing mechanisms for platforms under different environments (Rochet and Tirole (2003), Calliaud and Jullien (2003), Armstrong (2006), Armstrong (2007)). This paper is an extension of the Armstrong (2006), where it was concluded that there are three main factors that determine the price offered to the two groups by the platform –

Relative size of the cross group externalities: If an agent of group 1 provides a large positive externality to each agent of group 2, then the platform aggressively targets the agents of group 1. This large, positive cross group externality of group-1 agents intensify the competition among platform and, therefore, reduce platform's profit.

Fixed fees or per-transaction charges: If a platform charges on a lump sum basis, then the agent's payment does not explicitly depend on how well the platform performs on the other side of the market. However, in the case of a per-transaction charge, an agent's payment might be an explicit function of the effort of the platform in attracting agents of the other side. The main difference between the two charging methods is that the network effect is weaker in the latter case.

Single homing or multi-homing: When an agent joins only one platform, then we say that this agent is single homing. When an agent joins several platforms then he/she is said to be multi-homing. Now suppose a multi homing agent wants to interact with an agent who is single homing, then the multi homing agent has no choice but to deal with a platform of single homing agents. The platform with single homing agents has monopoly power over providing access to multi homing agents.

Armstrong (2006), however, neglects the possibility that agents also care about the number of agents from the same group who join the platform. Since an individual cares about the number of agents with whom he/she is competing, this 'congestion' is an important factor affecting the pricing mechanism and the profit of the platform. This paper assumes that an agent on a platform feels congestion when the number of agents from the same group exceed a proportion of the number of agents on the opposite side. That proportion is known as threshold or tolerance parameter. For example, in the case of cab aggregators, a rider may feel congestion when the number of riders using the same app to book a cab is higher than the number of drivers present on the other side of the same platform. In this case, congestion is reflected in higher expected time of arrival or surge pricing. Similarly from the view point of driver, if the number of drivers is more than number of riders, then the chances of getting a booking will become low for a given driver, or the price the rider pays will decrease.

This paper extends Armstrong's model by adding an extra term which capture the congestion effect (Armstrong, 2006). The utility function of an agent on the platform is not only dependent on the cross-group externalities and price charged by the platform, but also on the level of congestion in the platform. As discussed above, dis-utility from congestion will be function of both - the number of agents in the same group and the number of agents in the opposite group. This paper concludes that because of congestion, the network effect or cross group externalities increases, reducing the price setting power of platform, but the effective transportation cost of switching between platforms also increases, giving power to the platform to set a higher price (a conclusion in line with the Hotelling problem). These two opposing effects will determine the equilibrium price and profit for the platform, especially when agents on both side are single homing. In the single homing case, when tolerance parameter is small (less than one), congestion will fetch more profit for the platforms. However, in the multi-homing case, the condition under which congestion is beneficial for platforms is reversed and profit becomes higher when tolerance parameter is higher (greater than two).

Rochet and Tirole (2006) discuss the conditions under which a market can be classified as a two-sided market. It is concluded by them that the structure of the market is main determinant for two-sided market, instead of the pricing mechanism. Rochet and Tirole also show that the failure of the Coase theorem is a necessary but not a sufficient condition for a market to be classified as two-sided. Caillaud and Jullien (2003) first develop the model for single homing case with perfect competition where agents have no intrinsic preference for any platform (expect for the price charged by the platform). Thus, all the agents join that platform which offers lowest price and therefore, the efficient outcome involves all the agents using same platform and profit for that platform being zero. In contrast, this paper considers an explicit transportation cost which set intrinsic preferences of agents for the platforms. For multi-homing case, Caillaud and Jullien (2003) find that in equilibrium, the group which is multi-homing will be left with no surplus and the single homing side will be treated favourably.

There is some literature on congestion in two-sided markets, but mainly confined on the debate of net-neutrality. Cheng et al.(2011) capture the congestion in two-sided market by modelling the situation with and without preferential treatment of some content providers. When one content provider gets preferential treatment, then that content provider faces congestion only from the traffic of their own content-watchers. But other content providers face congestion because of the entire traffic. Cheng et al. find that if the principle of net-neutrality is abandoned, then the Internet Service Provider (ISP) will always benefit. And a content provider will be worse-off. Surplus of a consumer can be higher or lower depending upon the parameter used in the paper (e.g. Poisson arrival rate of content). Thus, overall social surplus can be higher or lower accordingly. Congestion captured in this paper is not specific to net-neutrality or for ISPs. This paper addresses congestion for the generic case. Another paper on congestion is a empirical work done on UBER surge pricing (Chen and Sheldon, 2015). Chen and Sheldon (2015) collect the data of randomly selected UberX partners for every trip in five different cities (Chicago, DC, Miami, San Diego and Seattle). They find that the drivers on UBER platform respond to surge pricing and surge price significantly increase the supply of drivers. The assumption of this work differs as the Chen and Sheldon (2003) assume that the driver quits once they hit a daily income target.

This paper is structured as follows. Section 2 will discuss the case of single homing and different cases under which price and profit is higher because of congestion. Section 3 will discuss the multi-homing case and construct a symmetric equilibrium for the case. Section 4 concludes the paper.

2 Competition in Two Sided Single Homing Platform

There are two competing platforms and here we assume that each agent choose to join single platform.

Suppose there are two groups of agents-1 and 2, and two platforms- A and B. Both platforms allow the two groups of agent on their respective platforms to interact. A member in each group cares about both - the number of agents in the other group and the number of agents of their own group on the platform. An agent in any group derives a positive utility from the presence of agents in the other group and negative utility from the agents in their own group. If agents of group 1 and 2 join the platform A, their utility will be (u_1^A, u_2^A) respectively. If platform A attracts n_1^A and n_2^A agents of the two groups, then the utilities they derive are -

$$u_1^A = \alpha_1 n_2^A - p_1^A - \beta_1(n_1^A - \lambda n_2^A)$$
$$u_2^A = \alpha_2 n_1^A - p_2^A - \beta_2(n_2^A - \lambda n_1^A)$$

where p_1^A and p_2^A are the prices charged by the platform A to the two groups. The parameter α_1 measures the benefit a group-1 agent derives from interacting with each group-2 agent, and α_2 measures the benefit a group-2 agent derives from interacting with each group-1 agent. The parameter β_1 is the measure of loss in the utility due to congestion i.e. as n_1 increases by unit one, with $\lambda * n_2$ constant, each agent of group-1 experiences disutility measured by β_1. Similarly, β_2 is the measure of loss due to congestion for group-2 agents. The parameter β is known as congestion parameter. λ is a tolerance or threshold parameter which can be interpreted as follows- for a given n_1^A and n_2^A in equation (1), as λ increases, disutility from congestion reduces. Thus, λ is the parameter which measures the tolerance of each agent for congestion. It is assumed that tolerance level for both groups is same. Thus, the utility functions for the two group of agents on platform A are:

$$u_1^A = (\alpha_1 + \beta_1\lambda)n_2^A - p_1^A - \beta_1 n_1^A \tag{1}$$
$$u_2^A = (\alpha_2 + \beta_2\lambda)n_1^A - p_2^A - \beta_2 n_2^A \tag{2}$$

Similarly, for the agents who join platform B, the utility function for the two groups of agents are:

$$u_1^B = (\alpha_1 + \beta_1\lambda)n_2^B - p_1^B - \beta_1 n_1^B \tag{3}$$

$$u_2^B = (\alpha_2 + \beta_2\lambda)n_1^B - p_2^B - \beta_2 n_2^B \tag{4}$$

Utility offered by platforms A and B to an agent of group 1 is (u_1^A, u_1^B), and similarly for an agent of group 2 is (u_2^A, u_2^B). It is assumed that the number of agents who join platform A is given by the Hotelling specification. The platforms are located at two endpoints and agents are uniformly located on $[0, 1]$.

$$n_1^A = \frac{1}{2} + \frac{u_1^A - u_1^B}{2t_1}; \qquad n_2^A = \frac{1}{2} + \frac{u_2^A - u_2^B}{2t_2} \tag{5}$$

where t_1, t_2 are the product differentiation parameters for the two groups that describe the competitiveness of two sides of the market. It is a concept similar to transportation cost in Hotelling problem. Another possible interpretation for it is- cost of signing up for a service, initial set up cost for learning about a new service or cost of reaching the platforms.

Now using Equation (1), (2), (3), (4) in equation (5) and the fact that $n_1^B = 1 - n_1^A$, we get following expression -

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \beta_1\lambda)(2n_2^A - 1) - (p_1^A - p_1^B) - \beta_1(2n_1^A - 1)}{2t_1}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \beta_2\lambda)(2n_1^A - 1) - (p_2^A - p_2^B) - \beta_1(2n_2^A - 1)}{2t_2} \end{aligned} \tag{6}$$

We can further simplify equation (6) to get the following -

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(2n_2^A - 1) - (p_1^A - p_1^B)}{2(t_1 + \beta_1)}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \beta_2\lambda)(2n_1^A - 1) - (p_2^A - p_2^B)}{2(t_2 + \beta_2)} \end{aligned} \tag{7}$$

Equation (7) says that one extra agent of group-2 on the platform will attract $\frac{(\alpha_1+\beta_1\lambda)}{(t_1+\beta_1)}$ additional agents of group-1 on the platform. Also, upon comparison of equation (7) with the case of no congestion (Armstrong model) i.e. β_1 and β_2 equal to 0, it can be concluded that the market power of platform increases from t_1,t_2 to $(t_1 + \beta_1)$,$(t_2 + \beta_2)$ and network effect increases from α_1, α_2 to $(\alpha_1 + \lambda\beta_1)$,$(\alpha_2 + \lambda\beta_2)$ for platforms A and B, respectively.

Given that platforms will charge the price pairs (p_1^A, p_2^A),(p_1^B, p_2^B), solving equations (6) or (7) simultaneously gives the following outcome:

$$\begin{aligned} n_1^A =& \frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(p_2^B - p_2^A) + (t_2 + \beta_2)(p_1^B - p_1^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}; \\ n_2^A =& \frac{1}{2} + \frac{(\alpha_2 + \lambda\beta_2)(p_1^B - p_1^A) + (t_1 + \beta_1)(p_2^B - p_2^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]} \end{aligned} \tag{8}$$

Assuming that each platform has a per-agent cost f_1 and f_2 for serving agents of group-1 and group-2, respectively, the profit of platform A is

$$\begin{aligned} \pi^A = (p_1^A - f_1)\Big[\frac{1}{2} + \frac{(\alpha_1 + \lambda\beta_1)(p_2^B - p_2^A) + (t_2 + \beta_2)(p_1^B - p_1^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}\Big] + \\ (p_2^A - f_2)\Big[\frac{1}{2} + \frac{(\alpha_2 + \lambda\beta_2)(p_1^B - p_1^A) + (t_1 + \beta_1)(p_2^B - p_2^A)}{2[(t_1 + \beta_1)(t_2 + \beta_2) - (\alpha_1 + \beta_1\lambda)(\alpha_2 + \beta_2\lambda)]}\Big] \end{aligned} \tag{9}$$

The necessary and sufficient condition for market sharing equilibria to exist is

$$4(t_1 + \beta_1)(t_2 + \beta_2) > ((\alpha_1 + \alpha_2) + \lambda(\beta_1 + \beta_2))^2 \tag{10}$$

This expression can be derived from the stability condition i.e. determinant of Hessian matrix is greater than zero.

From expression (9), the best response function of platform A's to platform B's price pair can be derived through first order condition. From expression (10), it can be concluded that there is no asymmetry. Thus, for the case of symmetric equilibrium, where each platform offers the same price pair (p_1, p_2), the first order conditions for equilibrium prices are

$$\begin{aligned} p_1 =& f_1 + (t_1 + \beta_1) - \frac{(\alpha_2 + \lambda\beta_2)}{(t_2 + \beta_2)}(\alpha_1 + \lambda\beta_1 + p_2 - f_2); \\ p_2 =& f_2 + (t_2 + \beta_2) - \frac{(\alpha_1 + \lambda\beta_1)}{(t_1 + \beta_1)}(\alpha_2 + \lambda\beta_2 + p_1 - f_1) \end{aligned} \tag{11}$$

If there is no cross group externalities or congestion, then in Hotelling model the equilibrium price would be $p_1 = f_1 + t_1$. Thus, all the term other than f_1 and t_1 is due to cross group externalities with congestion.

Now, on solving the two equations (11) simultaneously, the following can be derived:

Lemma 1. Suppose that the condition (10) holds, then, the congestion model with two-sided single homing has a unique equilibrium, and this equilibrium is symmetric. Equilibrium prices for group-1 and group-2 are given by

$$\begin{aligned} p_1 =& f_1 + (t_1 + \beta_1) - (\alpha_2 + \lambda\beta_2); \\ p_2 =& f_2 + (t_2 + \beta_2) - (\alpha_1 + \lambda\beta_1) \end{aligned} \tag{12}$$

Let the equilibrium price levels for group-1 and group-2 in case of no congestion be denoted by p_1^o and p_2^o, respectively. This is obtained by putting congestion parameters, β_1 and β_2, equal to zero. Obtained expression is

$$\begin{aligned} p_1^o =& f_1 + t_1 - \alpha_2; \\ p_2^o =& f_2 + t_2 - \alpha_1 \end{aligned} \tag{13}$$

This is the same as the equilibrium price level in Armstrong's model (2006). Upon comparing equations (12) and (13), it can be concluded that transportation cost increases from (t_1, t_2) to $((t_1 + \beta_1), (t_2 + \beta_2))$ for both groups because of congestion. Similarly network parameter increases from (α_1, α_2) to $((\alpha_1 + \lambda\beta_1), (\alpha_2 + \lambda\beta_2))$ in case of congestion. Increase in transportation cost in case of congestion is due to the fact that when an agent of group i joins a platform, the agent will not only incur a transportation cost t_i, but also get a dis-utility β_i due to congestion. So the effective transportation cost will become $(t_i + \beta_i)$. This increase in transportation cost will give more power to platforms to set a higher price. Increase in network effect is due to the fact that, in case of congestion, when an agent of group j joins a platform, he/she will not only increase the utility of the opposite group by α_i, but also decrease the disutility because of congestion by $\lambda\beta_i$. Thus, the effective network effect becomes $(\alpha_i + \lambda\beta_i)$. This increase in network effect reduces the power the platform to set higher prices.

From (9) and (12), in equilibrium each platform makes a profit equal to

$$\pi^* = \frac{(t_1 + \beta_1) + (t_2 + \beta_2) - (\alpha_1 + \lambda\beta_1) - (\alpha_2 + \lambda\beta_2)}{2}$$

Now, the profit expression in case of no congestion (Armstrong's model) i.e. β_1 and β_2 equal to zero is given by

$$\pi^o = \frac{(t_1 + t_2) - (\alpha_1 + \alpha_2)}{2}$$

So the condition when congestion is beneficial and fetches more profit for the platform is given by - $(\pi^* - \pi^o) > 0$.

Proposition 1. Suppose condition (10) holds and equilibrium price for both the groups is given by equation (12). Then condition under which platforms earn higher profit because of congestion is -

$$\lambda < 1$$

It means that the when tolerance parameter is less than one, it is profitable for the platform.

This result is intuitive since the market power of platforms to set higher price because of congestion increases by β_i (since transportation cost is increass to $(t_i + \beta_i)$ from t_i) and decreases by $\lambda\beta_i$ (network effect increases to $(\alpha_i + \lambda\beta_i)$ from α_i) for group i. Thus, the value of λ will determine the overall increase in market power of platform for each group and subsequently the profit for platform. The various cases that arise depending upon the possible value of λ are discussed below. Equation (12) can be re-written in following manner -

$$p_1 = f_1 + t_1 - \alpha_2 + (\beta_1 - \lambda\beta_2);$$

$$p_2 = f_2 + t_2 - \alpha_1 + (\beta_2 - \lambda\beta_1)$$

Upon comparing the above expression with equation (13), it is seen that $p_1 > p_1^o$ whenever $(\beta_1 - \lambda\beta_2) > 0$ or $\lambda < (\frac{\beta_1}{\beta_2})$. Therefore, when $\lambda < (\frac{\beta_1}{\beta_2})$, the equilibrium price charged by the platforms to group 1 is greater in case of congestion. Similarly when $(\beta_2 - \lambda\beta_1) > 0$ or $\lambda < (\frac{\beta_2}{\beta_1})$, equilibrium price charged to group 2 by the platform is higher in case of congestion.

Case 1. when $\lambda < min\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$

In this case, price charged to both the groups is higher because of congestion (i.e. $p_1 > p_1^o$ and $p_2 > p_2^o$). Since $\lambda < min\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$, it implies λ is less than one also. Thus, the platform earns more profit due to congestion ($\pi^* > \pi^o$). This is because the increment in the transportation cost (from t_i to $(t_i + \beta_i)$) is higher than the increase in network effect (which increase from α_i to $(\alpha_i + \lambda\beta_i)$ due to which platforms can set higher price for both the groups and earn higher profits in the equilibrium.

Case 2. when $\lambda > max\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$

In this case price charged to both the groups is lower because of congestion (i.e. $p_1 < p_1^o$ and $p_2 < p_2^o$). Since $\lambda > max\left[\left(\frac{\beta_1}{\beta_2}\right), \left(\frac{\beta_2}{\beta_1}\right)\right]$, it implies λ is also greater than one. Thus, the platform earns lesser profit due to congestion ($\pi^* < \pi^o$). This is because the increment in the transportation cost (from t_i to $(t_i + \beta_i)$) is lower than the increase in network effect (which increase from α_i to $(\alpha_i + \lambda\beta_i)$ due to which platform's power to set higher price for both the groups reduces and thus earn lower profits in the equilibrium in comparison to no-congestion case.

For the other two cases, let us assume without loss of generality that $\beta_1 > \beta_2$.

Case 3. when $\left(\frac{\beta_2}{\beta_1}\right) < \lambda < 1 < \left(\frac{\beta_1}{\beta_2}\right)$

In this case price charged to group-1 agents is higher and to group-2 agents is lower because of congestion ($p_1 > p_1^o$ and $p_2 < p_2^o$). Due to congestion, agents of group-2 get a subsidy at the cost of a higher price charged to group-1 (the group which has a higher congestion parameter have to pay a higher price). Since λ is less than one, the overall profit of both the platforms is higher because of congestion ($\pi^* > \pi^o$). Platforms extract more from group-1 and then it transfers the benefit to agents of group-2, due to which overall profit of platform rises.

Case 4. when $\left(\frac{\beta_2}{\beta_1}\right) < 1 < \lambda < \left(\frac{\beta_1}{\beta_2}\right)$

This case is similar to the previous case, with the only difference that the profit of both platforms is lower with congestion than without congestion, i.e., $p_1 > p_1^o$ and $p_2 < p_2^o$ but $\pi^* < \pi^o$. Unlike the previous case, the platform cannot extract more from group-1 agents and transfer the benefit to group-2 agents, due to which overall profit of platform decreases.

Remark: This is a static model and it is be difficult to capture dynamics of the congestion through this model. Nonetheless, some implications of dynamic game can be inferred from this analysis. Suppose tolerance parameter is greater than one initially. Then it is beneficial for platforms to reduce the tolerance parameter (and hence tolerance) of the agents on the platform. The only way for this is to give better and innovative service to all the agents. Over time, tolerance parameter (and hence tolerance) will reduce and platforms will start earning more profit by congestion. Thus, congestion will lead to better service and innovation. This is true for net-neutrality principle also. By abandoning net-neutrality principle, platforms start getting more profit immediately and there is very less effect of congestion on pricing and profit of the platform. Thus, abandoning of net-neutrality principle can seriously hamper innovation and urge to provide better service to the agents on the platform.

3 Competitive Bottlenecks: Multi-homing case

This section represent the case when multi-homing is possible. Multi-homing means an agent joins both the platforms. Now there are two cases to consider: (i) one group of agents is single home and other group is multi-home, and (ii) both the groups are multi-homes. If the main purpose of joining the platform is to derive a utility by interacting with the opposite group, then case (ii) will not arise, so only case (i) is discussed here. This is because if the opposite group is already a multi-home, then the agents of single homing group have no incentive to join the next platform. Thus, the group which is multi-home should not get any intrinsic utility just by joining any platform and agents of single homing group get intrinsic utility only once (duplicate intrinsic utility is not allowed). Before going into the details of multi-homing, the condition under which a group is always single homing is derived. An agent will join only one platform when the cost of joining other platform is higher than the utility it gets from joining another platform. The cost of joining the another platform for an agent of group i with congestion is $(t_i + \beta_i)$ (effective transportation cost in case of congestion) plus price charged by the platform, whereas the utility it gets by joining another platform is given by $(\alpha_i + \lambda\beta_i)$. So, whenever the price of joining a new platform is non-negative and $(t_i + \beta_i) > (\alpha_i + \lambda\beta_i)$, an agent will not join the other platform. Prices have to be non-negative for sufficient condition because when transportation cost is higher that utility derived, but platform pays (i.e negative price) for subscription then it is be possible that net utility becomes higher by joining one more platform. Non-negative restriction on price avoids that above case.

Without loss of generality it is assumed that agents of group-1 are single homing and group-2 agents are multi-homing. Thus, for group-1 $(t_1 + \beta_1) > (\alpha_1 + \lambda\beta_1)$, with non-negative price restriction, holds. For agents of group-2, transportation cost must be less than network effect. For simplicity it is assumed that $t_2 = 0$. Also, the non-negative restriction of price charged is put on the platform for both the groups. Thus, to model the multi-homing case, following assumptions are made:

A1. Intrinsic utility of joining the platform for agents of group-1 (say, u_1^o) is sufficiently high and intrinsic utility of group-2 agents (say, u_2^o) by joining any platform is zero. This assumption ensures that all the agents of group-1 will join at least one platform and agents of group-2 will join any platform only if net utility becomes higher by joining the platform.

A2. Price charged by the platforms from both the group must be non-negative.

A3. $(t_1 + \beta_1) > (\alpha_1 + \lambda\beta_1)$. A2 and A3 ensure that the group-1 is single homing. A3 can also be expressed as: $\lambda < 1 + \left(\frac{t_1 - \alpha_1}{\beta_1}\right)$

A4. $t_2 = 0$. This assumption ensures that every agent of group-2 will either join a platform or will not join. It does away with the case where some agents of group-2 join a given platform and some do not. When transportation cost (t_2) is zero, agents become homogeneous i.e. there is no difference within the group.

A5. $f_2 < \frac{t_1}{2}$. There are two benefits for platforms in serving the agents of group-2. First, revenue for the platforms will increase. Second, it will enable the platforms to compete better for agents of group-1. The benefit of an agent of group-2 joining the platform must outweigh the cost of serving the agent. A5 ensure this condition.

If assumptions A1 to A5 hold, then the agents of group-1 will always be single homing and there will be four possible configurations of group-2 to consider: (i) all agents of group-2 will multi-home, (ii) all agents of group-2 will single home on platform A, (iii) all agents of group-2 will single home on platform B, and (iv) all agents of group-2 do not join any platform. Here it is also assumed that, when an agent of group-2 is indifferent between joining or not joining any platform, then (s)he will join the platform.

Configuration 1. All agents of group-2 will multi-home.

Since every agent of group-2 joins both the platform, $n_2^A = n_2^B = 1$, and the utility function of an agent of group-1 upon joining platform A is given by-

$$u_1^A = \alpha_1(n_2^A) - p_1^A - \beta_1(n_1^A - \lambda(n_2^A)) + u_1^o$$

$$u_1^A = \alpha_1(1) - p_1^A - \beta_1(n_1^A - \lambda(1)) + u_1^o$$

Above can also be written in following manner:

$$u_1^A = (\alpha_1 + \beta_1\lambda) - p_1^A - \beta_1 n_1^A + u_1^o$$

Similarly,

$$u_1^B = (\alpha_1 + \beta_1\lambda) - p_1^B - \beta_1 n_1^B + u_1^o$$

As in the previous section, it is assumed that the platforms are located at two endpoints of $[0,1]$ and agents are distributed uniformly on $[0,1]$. It is also assumed that the number of agents of group-1 on any platform is given by the Hotelling specification. Thus, the number of agents of group-1 who will join platforms A and B is given by:

$$\begin{aligned} n_1^A &= \frac{1}{2} + \frac{p_1^B - p_1^A}{2(t_1 + \beta_1)}; \\ n_1^B &= \frac{1}{2} + \frac{p_1^A - p_1^B}{2(t_1 + \beta_1)} \end{aligned} \tag{14}$$

where (p_1^A, p_1^B) are the prices charged by the platforms A and B to the agents of group 1.

Now, the utility of an agent of group 2 (who joins both the platforms) is given by -

$$u_2^{AB} = \alpha_2(n_1^A + n_1^B) - p_2^A - p_2^B - \beta_2(n_2^A - \lambda n_1^A) - \beta_2(n_2^B - \lambda n_1^B)$$

$$u_2^{AB} = \alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda)$$

Agents of group-2 will multi-home if following conditions hold:

Condition 1. Agents of group-2 should prefer multi-homing in comparison to not joining any platform. That is, the utility derived by the group-2 agents by multi-homing must be non-negative.

$$u_2^{AB} = \alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda) \geqslant 0$$

$$p_2^A + p_2^B \leqslant (\alpha_2 + \beta_2\lambda) - 2\beta_2$$

Condition 2. Group-2 agents prefer multi-homing to single homing on platform A. This happens only when the agents of group-2 get higher utility in case of multi-homing.

$$\alpha_2 - p_2^A - p_2^B - \beta_2(2 - \lambda) \geqslant \alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A)$$

$$p_2^B \leqslant (\alpha_2 + \beta_2\lambda)n_1^B - \beta_2 \tag{15}$$

Condition 3. Group-2 agents prefer multi-homing to single homing at platform B. Same as above, utility in case of multi-homing should be higher (or equal) to utility in the case of multi-homing.

$$\alpha_2 - p_2^A - p_2^B - \beta_2(2-\lambda) \geqslant \alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B)$$

$$p_2^A \leqslant (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2 \tag{16}$$

It can be confirmed that condition 1 is satisfied when conditions 2 and 3 hold.

The profit function of the platforms A and B are:

$$\pi^A = (p_1^A - f_1)\Big[\frac{1}{2} + \frac{p_1^B - p_1^A}{2(t_1+\beta_1)}\Big] + (p_2^A - f_2) \tag{17}$$

$$\pi^B = (p_1^B - f_1)\Big[\frac{1}{2} + \frac{p_1^A - p_1^B}{2(t_1+\beta_1)}\Big] + (p_2^B - f_2) \tag{18}$$

Configuration 2. All agents of group-2 will join only platform A.

Since group-2 agents only join platform A, $n_2^A = 1; n_2^B = 0$. The proportion of group-1 agents who join platform A is given by-

$$n_1^A = \frac{1}{2} + \frac{(p_1^B - p_1^A) + (\alpha_1 + \lambda\beta_1)}{2(t_1+\beta_1)}$$

All agents of group-2 will join only platform A if the following conditions hold:

Condition 1. Group-2 agents prefer to join platform A in comparison to not joining any platform:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant 0$$

$$p_2^A \leqslant (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2$$

Condition 2. Group-2 agents prefer to single home on platform A in comparison to multi-home:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant \alpha_2 - p_2^A - p_2^B - \beta_2(1-\lambda)$$

$$p_2^B \geqslant (\alpha_2 + \beta_2\lambda)n_1^B$$

Condition 3. Group-2 agents prefer to single home on platform A instead of platform B:

$$\alpha_2(n_1^A) - p_2^A - \beta_2(1 - \lambda n_1^A) \geqslant \alpha_2(1 - n_1^A) - p_2^B - \beta_2(-\lambda n_1^B)$$

$$p_2^B - p_2^A \geqslant (\alpha_2 + \beta_2\lambda)(1 - 2n_1^A) + \beta_2$$

Condition 3 follows from conditions 1 and 2.

The profit functions of the platforms A and B are:

$$\pi^A = (p_1^A - f_1)\Big[\frac{1}{2} + \frac{p_1^B - p_1^A + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\Big] + (p_2^A - f_2)$$

$$\pi^B = (p_1^B - f_1)\Big[\frac{1}{2} + \frac{p_1^A - p_1^B - (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\Big]$$

Configuration 3. All agents of group-2 will only join platform B.

By symmetry $n_2^A = 0; n_2^B = 1$ and proportion of group-1 agent on platform B is given by:

$$n_1^B = \frac{1}{2} + \frac{(p_1^A - p_1^B) + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}$$

All agents of group-2 will join only platform B if following conditions hold:

Condition 1. Group-2 agents prefer to join platform B in comparison to not joining any platform:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant 0$$

$$p_2^B \leqslant (\alpha_2 + \beta_2\lambda)n_1^B - \beta_2$$

Condition 2. Group-2 agents prefer to single home on platform B instead of multi-home:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant \alpha_2 - p_2^B - p_2^A - \beta_2(1 - \lambda)$$

$$p_2^A \geqslant (\alpha_2 + \beta_2\lambda)n_1^A$$

Condition 3. Group-2 agents prefer to single home on platform B in comparison to single home on platform A:

$$\alpha_2(n_1^B) - p_2^B - \beta_2(1 - \lambda n_1^B) \geqslant \alpha_2(1 - n_1^B) - p_2^A - \beta_2(-\lambda n_1^A)$$
$$p_2^A - p_2^B \geqslant (\alpha_2 + \beta_2\lambda)(1 - 2n_1^B) + \beta_2$$

The profit functions of the platforms A and B are:

$$\pi^A = (p_1^A - f_1)\Big[\frac{1}{2} + \frac{p_1^B - p_1^A - (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\Big]$$

$$\pi^B = (p_1^B - f_1)\Big[\frac{1}{2} + \frac{p_1^A - p_1^B + (\alpha_1 + \lambda\beta_1)}{2(t_1 + \beta_1)}\Big] + (p_2^B - f_2)$$

Configuration 4. All agents of group-2 do not join any platform.

The proportion of agents of group-1 who join platforms A and B is the same as equation (14). This configuration is derived from the Hotelling problem for agents of group-1 where pricing for group-1 is given by -

$$p_1^A = p_1^B = f_1 + t_1 + \beta_1$$

This configuration is realized only when all the inequalities of configuration 1 are reversed.

Remark: For a range of prices; there is more than one configuration is possible. For example: suppose both the platform offer same pair of prices (p_1, p_2). If $p_2 \leqslant \frac{(\alpha_2+\lambda\beta_2)}{2} - \beta_2$, then configuration 1 is consistent. If in addition to above condition; p_2 also satisfies the condition $p_2 \geqslant (\alpha_2 + \lambda\beta_2)\Big(\frac{1}{2} - \frac{(\alpha_1+\lambda\beta_1)}{2(t_1+\beta_1)}\Big)$, then configurations 1 and 2, both, are consistent. Thus agents have to coordinate their choices for a given price pair.

Now to deal with the issue of multiplicity of consistent demand configurations, certain restrictions have to be imposed. As in Armstrong and Wright (2007), this paper also imposes the following two restrictions on the range of possible equilibria.

1. Inertia Condition: This condition states that the equilibrium must be robust to the deviation or change in price set by the platform, provided the change in price does not result in a change in configuration, i.e., the original configuration remains consistent with changed price. If the configuration changes, then the new configuration will require simultaneous coordination of agents to move.

Thus, the inertia condition implies that if it is difficult for agents to coordinate to move to a new configuration, the equilibrium price should be such that no deviation is possible.

2. Monotonicity Condition: According to this condition, if a platform deviates from the equilibrium price pair such that (weakly) reduces both the prices, then it must (weakly) increase that platform's demand from both the groups.

Given these two conditions (or restrictions), the equilibrium pricing for both the platforms is constructed. If the inertia condition holds, the equilibrium price set by both the platforms is such that group-2 is left with no surplus.

Proposition 2. Let assumptions A1, A3 and A4 hold and suppose that the inertia condition applies. Then in equilibrium, agents of group-2 will be left with no surplus.

Proof: The following is a proof for configuration 1. Similar logic can be applied to other configurations as well. Let the equilibrium price pair for group-2 be (p_2^A, p_2^B). Now, consider by contradiction, that the agents of group-2 are left with some surplus in equilibrium. This means that the first condition of configuration-1 does not satisfy with equality (this condition is obtained by the condition $u_2^{AB} \geqslant 0$). Either inequality (15) or (16) does not satisfy with equality. Thus, from the expressions (17) and (18), it is possible for the platform to increase the profit until the expression (15) and (16) are satisfied with equality. Inertia condition ensure that we remain in configuration 1 scenario. However, this cannot be equilibrium price. At equilibrium, the expressions (15) and (16) satisfy with equality and group-2 agents are not left with any surplus. $\square$

Now to construct a symmetric equilibrium, configurations 2 and 3 are not considered, and only configuration 1 or 4 will remain. From the point of view of two-sided markets, however, configuration-4 is uninteresting (in fact pricing mechanism is already mentioned previously). Thus, focus is mainly on configuration-1 for symmetric equilibrium. By proposition 2, any equilibrium will need expressions (15) and (16) to hold with equality. And in case of symmetric equilibrium, price charged to group-2 will be given by -

$$p_2 = p_2^A = p_2^B = \frac{(\alpha_2 + \lambda\beta_2)}{2} - \beta_2$$

or,

$$p_2 = \frac{\alpha_2 + (\lambda - 2)\beta_2}{2}$$

where (p_1, p_2) is a symmetric equilibrium price pair set by both the platforms. If platform B sets the price $(p_1, \left(\frac{\alpha_2+(\lambda-2)\beta_2}{2}\right))$ in equilibrium, then platform A can deviate if deviation is profitable. Thus if p_1 is equilibrium price for group-1, then it has to be in a manner such that any deviation from p_1 is non-profitable for both the platforms.

The fact that zero surplus is left with group-2 in equilibrium reduces the possible set of equilibria significantly. The reason is that that suppose platform A undercuts the price for group-1 slightly, then by monotonicity condition this slight undercut will attract more agents of group-1 on platform A (and hence fewer on platform B). Since initially the utility of agents of group-2 is zero on both the platforms, a slight undercut of price by platform A for group-1 agent will result in negative utility for group-2 agents on platform B. Thus, a small undercut in price for group-1 agent by platform A will result in group-2 agents joining platform A exclusively. That is, a small reduction in price of group-1 leads to a movement from configuration-1 to configuration-2. But in case of symmetric equilibrium, both the platforms undercut the price of subscription for group-1. Thus, the reduction in p_1^A or p_1^B is profitable only when the equilibrium price,p_1, is greater than marginal cost of serving of group-1 agent. That is, deviation is profitable only when $p_1 > f_1$.

Lemma 2. When $\alpha_1, \alpha_2 > 0$ and monotonicity condition holds in addition to assumptions A1, A3 and A4, then any symmetric equilibrium price for single homing side will be below the cost of serving, i.e., $p_1 \leqslant f_1$.

Another possible deviation available for platform A is to increase the price of group-1 agents (p_1^A) and reduce the price for group-2 (p_2^A) such that configuration-1 remains feasible. Given the inertia condition, this deviation will not result in a change in configuration. For (p_1, p_2) to equilibrium, there does not exist such deviation which will increase the profit of platform A. That is, derivative of profit expression with respect to p_1^A at ($p_1^A = p_1$) should be non-positive.

$$
\begin{aligned}
\pi^A &= (p_1^A - f_1)n_1^A + (p_2^A - f_2) \\
\pi^A &= (p_1^A - f_1)n_1^A + (\alpha_2 + \beta_2\lambda)n_1^A - \beta_2 - f_2 \\
\pi^A &= (p_1^A - f_1 + (\alpha_2 + \beta_2\lambda))\Big(\frac{1}{2} + \frac{p_1 - p_1^A}{2(t_1 + \beta_1)}\Big)
\end{aligned}
$$

Derivative of π^A with respect to p_1^A at ($p_1^A = p_1$) is given by:

$$
\frac{\partial \pi^A}{\partial p_1^A}\Big|_{p_1^A = p_1} = \frac{1}{2} - \frac{(p_1 - f_1 + (\alpha_2 + \lambda\beta_2))}{2(t_1 + \beta_1)}
$$

The above expression has to be non-positive for p_1 to be symmetric equilibrium price. Thus,

$$
p_1 \geqslant f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)
$$

For the existence of p_1 another restriction is required.

A6. $t_1 + \beta_1 \leqslant (\alpha_2 + \lambda\beta_2)$. This assumption ensures that the expression $f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)$ is less than f_1, so that the equilibrium value of p_1 exists.

The symmetric equilibria for configuration-1 can now be defined as follows.

Proposition 3. Assume A1 to A6 hold and $\alpha_1, \alpha_2 > 0$. Then there exists a range of symmetric equilibrium prices (p_1, p_2), such that $p_2 = \frac{\alpha_2 + (\lambda - 2)\beta_2}{2}$ and p_1 lies in the interval given by -

$$
\max\{0, f_1 + t_1 + \beta_1 - (\alpha_2 + \lambda\beta_2)\} \leqslant p_1 \leqslant f_1
$$

Group-1 is single homing and group-2 is multi homing.

The profit of both platforms A and B will be equal in equilibrium and is given by:

$$
\pi = \pi^A = \pi^B = \frac{(p_1 - f_1)}{2} + (p_2 - f_2)
$$

From proposition 3, it is clear that $(p_1 - f_1) \leqslant 0$, thus, the maximum value of profit for the platform is accrued when $p_1 = f_1$. And maximum profit is given by -

$$
\pi^{max} = \frac{\alpha_2 + (\lambda - 2)\beta_2}{2} - f_2
$$

Expression for maximum profit without congestion, i.e., ($\beta_2 = 0$) is given by:

$$\pi^o = \frac{\alpha_2}{2} - f_2$$

So the condition when congestion is beneficial (fetchs more profit) for the platform is given by: $\pi^{max} - \pi^o > 0$.

Proposition 4. Suppose assumptions A1 to A6 hold and pricing at equilibrium is given by proposition 3, then the condition under which platforms earn more profit because of congestion is given by -

$$\lambda > 2$$

In case of competitive bottleneck with congestion, a higher tolerance parameter implies a higher profit for the platforms. In this case, platforms do not earn any profit from group-1 agents and all the profit comes from group-2 agents. And the decision of group-2 agents to join any platform is independent of whether they already join any platform or not. Therefore, if tolerance of agents of group-2 is high, utility earned by group-2 agents will also be higher and the platform can extract a higher profit. When $\lambda > 2$, the effect of congestion on profit of platform gets reversed.

Remark: From A3, we know that $\lambda < 1 + \left(\frac{t_1 - \alpha_1}{\beta_1}\right)$. If $\left(\frac{t_1 - \alpha_1}{\beta_1}\right) < 1$, then $\lambda < 2$ always. It means when $t_1 < (\alpha_1 + \beta_1)$, then congestion will always reduce the profits of platform. Therefore, $t_1 > (\alpha_1 + \beta_1)$ is a necessary condition for congestion to improve the profit of platforms. But it is not a sufficient condition.

4 Conclusion

By incorporating congestion into the utility function of agents, cross group externalities (effective network effect) and effective transportation cost both increase. In case of single homing, it is concluded that when tolerance parameter is less than one, overall profits of both the platforms is higher because of congestion. Unlike single-homing case, in multi-homing there exists a non-symmetric equilibrium where all the agents of one group join only one platform. This paper finds out a symmetric equilibrium for multi-homing. Condition under which congestion is profitable for platform get reversed in multi-homing case. Now when tolerance parameter is greater than two, then profits of both the platform is increased due to congestion. This paper offers a static analysis, and it is difficult to capture overall dynamics of congestion on the platform. With time, tolerance parameter of agents change, so does the overall profit of the platforms. Thus, one plausible extension of this work is to make tolerance parameter dynamic in nature instead of static.

5 Reference

Armstrong, M. (2006). Competition in two-sided markets. The RAND Journal of Economics, 37(3), 668-691.

Armstrong, M., Wright, J. (2007). Two-sided markets, competitive bottlenecks and exclusive contracts. Economic Theory, 32(2), 353-380.

Chen, M. K., Sheldon, M. (2016, July). Dynamic Pricing in a Labor Market: Surge Pricing and Flexible Work on the Uber Platform. EC (p. 455).

Cheng, H. K., Bandyopadhyay, S., Guo, H. (2011). The debate on net neutrality: A policy perspective. Information systems research, 22(1), 60-82.

Rochet, J. C., Tirole, J. (2003). Platform competition in two-sided markets. Journal of the european economic association, 1(4), 990-1029.

Rochet, J. C., Tirole, J. (2006). Two-sided markets: a progress report. The RAND journal of economics, 37(3), 645-667.