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**Abstract**

*In this paper we introduce the technique of spectral envelope into the analysis of decoupling problem in Economics. Decoupling refers to the phenomenon that the business cycles in emerging market economies are de-synchronized from cyclical movements in the advanced economies. The analysis shows that all the countries in the sample have a common cycle of approximately 42 months. The results point to a strong possibility of a common global cycle of a periodicity between 3 to 4 years.*

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# The Spectral Envelope: An Application to the Decoupling Problem in Economics[1]

**Dilip M. Nachane[2], Amlendu Dubey[3]**

## Abstract

In this paper we introduce the technique of spectral envelope into the analysis of decoupling problem in Economics. Decoupling refers to the phenomenon that the business cycles in emerging market economies are de-synchronized from cyclical movements in the advanced economies. The analysis shows that all the countries in the sample have a common cycle of approximately 42 months. The results point to a strong possibility of a common global cycle of a periodicity between 3 to 4 years.

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## 1. Introduction

In signal processing, the power spectrum is routinely employed to study the salient features of the underlying signal, including most importantly its periodicities. But owing to leakage effects in window spectrum estimation, there are likely to be errors in frequency, amplitude and phase estimation (see Oppenheim et al (1999, p. 268-271), Ming and Kang (1996) etc.). The search is continuously on then, for more powerful methods to detect periodic features in observed time series. Among these, the *spectral envelope* approach seems particularly appealing.

The envelope of the *magnitude short-time spectrum* of a signal is often termed the *spectral envelope* and the concept has seen extensive use in sound analysis and synthesis, speech processing, satellite data analysis, biomedical engineering etc. (see Beauchamp (2005), Li and Turek (1996), Ogawa and Lee (1990) etc.). In an important paper, Lagunas et al (1989),

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formalize the *spectral envelope* concept via the introduction of the so-called *analytic spectrum.*[4] Estimation of the spectral envelope so defined, is usually done either via *cepstrum analysis* or via *linear predictive coding* based on an all-pole filter transfer function (see Cadzow (1982), Schwartz (1998) etc.).

The idea of the spectral envelope has been refined and generalized in a series of recent papers by Stoffer and his associates (see Stoffer et al (1993, 2000), McDougall et al (1997), Stoffer (1999) etc.). Their methods share some similarities with principal components analysis and projection pursuit regression. Just as these methods look for the linear combinations of the variables that best explain the total variability of the data, the spectral envelope approach of Stoffer et al, searches over the space of a general class of transformations that account for the maximum variation in the data. The spectral envelope method of Stoffer et al enjoys several advantages over the more traditional methods including most importantly (i) the class of transformations can be quite general and not restricted to be linear[5] (ii) the transformations can vary over time and frequency (iii) the method is better able to replicate the basic features of the data and (iv) and it can be extended without much difficulty to the case of multivariate time series for detecting common periodic features.

This paper seeks to apply the spectral envelope approach (Stoffer et al version) to an economic problem viz. the so-called *decoupling problem*, which has acquired importance in recent years with increasing globalization and the emergence of a cluster of rapidly growing EMEs (emerging market economies). The paper is written in a slightly expository style, with a view to highlighting the applicability potential of this approach to economics.

The introduction is followed by a brief discussion of definitional issues of the spectral envelope in Section 2. We then turn to a step-by-step description of the estimation of the spectral envelope in Section 3. Section 4 then indicates how the spectral envelope approach can be used to detect common peaks in multiple time series. Section 5 is devoted to an overview of the decoupling problem in economics, while the results of the spectral envelope application to the decoupling problem are discussed in Section 6. Finally, conclusions are gathered in Section 7, which also indicates some future areas of research.

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4 The analytic spectrum is discussed in detail in in Nadeu (1990) and Naidu (1995, p. 234).
[5] The conditions on the transformations are detailed in McDougall et al (1997, p. 200).

## 2. Spectral Envelope : Definitional Issues

In this section, we concentrate on the theoretical underpinnings of the spectral envelope approach. Let $\boldsymbol{Y(t)} = [Y_1(t), \ldots . Y_p(t)]$ be a p-dimensional vector-valued time series. Let $\mathcal{H}$ be a class of transformations from the p-dimensional Euclidean real space $\mathbb{R}^p$ to the real line $\mathbb{R}$ and let $\boldsymbol{g} = (g_1, \ldots . g_p)$ be a basis for this space satisfying

(i) $E\left[g_i\left(Y_j(t)\right)\right] = 0$ ; $\forall\, i, j = 1 \ldots p$

(ii) $E\left[g_i\left(Y_j(t)\right)^2 < \infty\right]$ ; $\forall\, i, j = 1 \ldots p$

(iii) The spectral density matrix $\boldsymbol{f}_{\boldsymbol{g}(\boldsymbol{Y})}(\omega)$ $of$ $\boldsymbol{g}(\boldsymbol{Y}(\boldsymbol{t}))$ exists ∀ ω and is continuous, where $\boldsymbol{g}(\boldsymbol{Y}(\boldsymbol{t})) = [\boldsymbol{g_1}(\boldsymbol{Y_1}(\boldsymbol{t}), \ldots . \boldsymbol{g_p}(\boldsymbol{Y_p}(\boldsymbol{t})]$ is a (px1) column vector.

The scaled time series w.r.t. the set $\mathcal{H}$ is defined as the linear combination.

$X_t(\beta) = \beta' \boldsymbol{g}(\boldsymbol{Y}(\boldsymbol{t}))$ where $\boldsymbol{\beta} = (\beta_1, \ldots \beta_p)$ is a (px1) column vector in $\mathbb{R}^p$.

Under the assumption (iii) above, the spectral density matrix $f_X(\omega, \beta)$ of $X_t(\beta)$ exists and is continuous.

**Def. 1:** Define the scalar $\lambda(\omega)$ by

$$\lambda(\omega) = \sup_{\beta \neq 0} \left\{\frac{f_X(\omega,\beta)}{\beta' V \beta}\right\} \tag{1}$$

where $V = var(g(\boldsymbol{Y}(\boldsymbol{t})))$ is assumed positive definite.

Then $\lambda(\omega)$ , $0 \leq \omega \leq \frac{1}{2}$ is referred to as the *spectral envelope* of the time series $\boldsymbol{Y(t)}$ **w.r.t.** to the class of transformations $\mathcal{H}$. The vector $\beta$ for which $\lambda(\omega)$ attains the supremum is called the *optimum scaling*.

The above definition is quite general but introduces an element of arbitrariness, since the spectral envelope varies with the selected class of transformations $\mathcal{H}$ . For applications, since no natural choice of transformations is available, Stoffer et al (1993, 2000) opt for the class of linear transformations (see Tiao et al (1993)).

Once the class of transformations $\mathcal{H}$ is restricted to be linear, the spectral envelope concept lends itself easily to applications. In the linear case, $X_t(\beta) = \beta' \boldsymbol{Y(t)}$ and the spectral envelope $\lambda(\omega)$ is the *largest* real eigenvalue of the determinantal equation

$$\left|\boldsymbol{f_Y}(\omega) - \lambda V_Y\right| = 0 \qquad (2)$$

and the optimal scaling/transformation is the corresponding eigenvector (see Stoffer (1990, p. 1343). Here $V_Y$ is the var-cov matrix and $\boldsymbol{f_Y}(\omega)$ is the spectral density matrix of $\boldsymbol{Y(t)}$

*It is important to note that the spectral envelope as well as the optimal scaling depend on the particular frequency ω being considered.*

## 3. <u>Spectral Envelope : Estimation</u>

Consider p *stationary* mean-adjusted real-valued time series[6] $Y_1(t), Y_2(t) \ldots . Y_p(t)$ ; t=1…n. Let $\sigma_j^2 = var\left(Y_j(t)\right), j = 1 \ldots p$ and define the *diagonal matrix* $\boldsymbol{V} = diag\{\sigma_1^2, \sigma_2^2 \ldots \sigma_p^2\}$. We next put

$$\mathbf{Z}(\boldsymbol{t}) = \boldsymbol{V^{-1/2} Y(t)} \qquad (3)$$

where $\boldsymbol{Y(t)} = \left[Y_1(t), \ldots \ldots Y_p(t)\right]$ is a (px1) column vector

Let $\boldsymbol{f_Y}(\omega)$ denote the spectral density matrix of $\boldsymbol{Y(t)}$ at frequency ω where ω takes the successive values $\omega_j = \left(\frac{j}{n}\right)$ ; $j = 1, 2 \ldots [\frac{n}{2}]$ (Here [k] denotes the greatest integer ≤ k). In general $\boldsymbol{f_Y}(\omega)$ will be a complex quantity. We denote its real part by $\boldsymbol{f_Y^{RE}}(\omega)$.

Similarly, if we let $\boldsymbol{f_Z}(\omega)$ denote the spectral density matrix of $\boldsymbol{Z(t)}$ at frequency ω, then

$$\boldsymbol{f_Z}(\omega) = \boldsymbol{V^{-1/2} f_Y^{RE} V^{-1/2}} \qquad (4)$$

[6] We assume the p series to be of equal length but if they are of unequal length, they can be padded to a common length which we call n. If any of the $Y_j(t)$, j=1…p are non-stationary then they must be rendered stationary by first removing unit roots and then eliminating trends. After the series have been rendered stationary, the sample means are removed from each series.

Let $\lambda(\omega)$ $and$ $\boldsymbol{\beta}(\omega)$ denote the largest (real-valued) eigenvalue and the associated eigenvector of the matrix $\boldsymbol{f_Z}(\omega)$ given by (4). The plot of $\lambda(\omega)$ against $\omega$ constitutes the *spectral envelope* for our problem and $\boldsymbol{\beta}(\omega)$ the optimal transformation/scaling**.**

The estimation of the spectral density matrix $\widetilde{\boldsymbol{f_Z}(\omega)}$ of $\boldsymbol{f_Z}(\omega)$ has to be done with some care.

Define the discrete Fourier transform (DFT) of the series $Y_j(t)$ by

$$dY_j(\omega) = \left(\frac{1}{\sqrt{n}}\right)\sum_{t=1}^{n} Y_j(t)exp\,(-2\pi it\omega) \qquad (5)$$

and let $\boldsymbol{dY}(\omega) = \left[dY_1(\omega), \ldots\ldots dY_p(\omega)\right]$ (a px1 column vector)

Further let $\boldsymbol{d^*Y(\omega)}$ be the complex conjugate column vector of $\boldsymbol{dY(\omega)}$ and define the periodogram $\boldsymbol{I(\omega)}$ of $\boldsymbol{Y(t)}$ **as**

$$\boldsymbol{I(\omega)} = \boldsymbol{dY(\omega)d^{*}Y(\omega)'} \qquad (6)$$

As is well-known the periodogram is an inconsistent estimate of the spectrum (see Koopmans (1995, p. 266)). A consistent estimate of the spectral density matrix $\boldsymbol{f_Y}(\omega)$ can, however, be obtained by using the weighted periodogram

$$\widehat{\boldsymbol{f_Y}}(\boldsymbol{\omega_k}) = \sum_{r=-m}^{m} h_r I(\omega_{k+r}) \qquad (7)$$

where $\omega_k = \left(\frac{k}{n}\right)$ is the k-th Fourier frequency , the $h_r$ are suitably chosen weights and the quantity m is the truncation parameter .

While a wide choice of weighting schemes (windows) is available such as the Bartlett, Parzen, Daniell, Priestley etc. (see the discussion in Priestley (1981, p. 432-449), Stoffer (1999, p. 1349) suggests the following symmetric triangular window as particularly convenient

$$h_0 = \left(\frac{1}{m+1}\right), h_j = \left(\frac{m+1-j}{(m+1)^2}\right),\ \ with\ h_{-j} = h_{+j}\ and\ j = 1 \ldots m \qquad (8)$$

The choice of the truncation parameter *m* depends on the length *n* of the observed time series. For shorter span of observations in the region of *n=100 to 200*, a choice of *m=7* seems reasonable (see Stoffer (op. cit.)).

Once $\widehat{\boldsymbol{f_Y}}(\boldsymbol{\omega_k})$ has been estimated in the above manner, we can obtain a consistent estimate of $f_Z(\omega)$ from (4) as

$$\widehat{\boldsymbol{f_Z}}(\boldsymbol{\omega_k}) = \boldsymbol{V}^{-1/2} \widehat{\boldsymbol{f}_{\boldsymbol{Y}}^{\boldsymbol{RE}}}(\boldsymbol{\omega_k}) \boldsymbol{V}^{-1/2} \tag{9}$$

## 4. Detecting Common Periodicities In Multiple Time Series

The *spectral envelope* introduced above, provides a convenient device for identifying common periodicities in a collection of multiple time series. As in section 3 above, we consider p *stationary* mean-adjusted real-valued time series $Y_1(t), Y_2(t) \dots . Y_p(t)$ ; t=1…n. Retaining the other notation as in the previous sections, we plot the graph of the spectral envelope $\lambda(\omega)$, and identify its various peaks. To determine whether a particular peak $\omega_0$ in the spectral envelope graph $\lambda(\omega)$, is significant, Stoffer (1999) suggests the following test.

Define $v_m^2 = \frac{1}{\sum_{r=-m}^{m} h_r^2}$ (10)

where the quantities $h_r$ $and$ $m$ are as defined in (8) above.

To test whether at $\omega_0$ , there is a significant peak in $\lambda(\omega)$ we examine whether or not

$$\lambda(\omega_0) > \left(\frac{2}{n}\right) exp\left(\frac{z_\alpha}{v_m}\right) \tag{11}$$

Here n is the common length of all the series and $z_\alpha$ is the upper tail value of the Standard normal distribution for level of significance α.

If the peak at $\omega_0$ is significant, the possibility is indicated that some of the time series in the collection may share a common periodicity at the frequency $\omega_0$ or in its small neighbourhood. It is possible to identify how many of the p series share a common periodicity at $\omega_0$ (or in its small neighbourhood). This requires us to test for the significance of the individual components of $\boldsymbol{\beta}(\omega_0)$ (the eigenvector corresponding to $\omega_0$ of the matrix $\boldsymbol{f_Z}(\omega)$ given by (4)). This may be done as follows. Arrange the components of $\boldsymbol{\beta}(\omega_0)$ in descending order of magnitude, renumbering them if necessary. (Note that this re-arrangement will vary from peak to peak and has to be done separately for each peak).

Suppose we wish to test the hypothesis that a particular component (say the last for specificity) of $\boldsymbol{\beta}(\omega_0)$ is zero. We now introduce a new vector $\widehat{\boldsymbol{\beta}(\omega_0)}$ which has the last component set at zero and the (k-1) components the same as that of $\boldsymbol{\beta}(\omega_0)$ but normalized so that their squares sum to unity i.e. if $\widehat{\beta_j}$ denotes the j-th component of $\widehat{\boldsymbol{\beta}(\omega_0)}$, then

$$\sum_{j=1}^{k-1} \widehat{\beta_j^2} = 1; \, and \, \widehat{\beta_k} = 0, \qquad (12)$$

Define $H(\omega) = \, \boldsymbol{f}_{\boldsymbol{Y}}^{\boldsymbol{RE}}(\omega) - \, \lambda(\omega_0)\mathrm{I}$ (13)

where I is (pxp) *identity* matrix

Introduce the (px1) vector,

$$\xi(\omega_0) = \sqrt{2}\, \upsilon_m \left[\boldsymbol{f}_{\boldsymbol{Y}}^{\boldsymbol{RE}}(\omega_0)\right]^{-\frac{1}{2}} H(\omega_0)\{\hat{\beta}(\omega_0) - \beta(\omega_0)\}\left(\frac{1}{\sqrt{\lambda(\omega_0)}}\right) \qquad (14)$$

And further define the scalar quantity $\psi$ as

$$\psi = \xi'^{(\omega_0)}\xi(\omega_0) \qquad (15)$$

Under the null hypothesis that $\widehat{\beta_k} = 0$, it can be shown that as $m, n \, \rightarrow \infty$, the quantity $\psi$ in (15) approaches in distribution to a distribution $\mathbb{Z}(\psi)$ such that

$$\chi_{p-1}^2 \; \leq \mathbb{Z}(\psi) \leq \; \chi_{p-2}^2 \qquad (16)$$

(see Stoffer (1999, p. 1346 )

(16) can be used to test for the presence of common periodicities at a particular frequency $\omega_0$, by successively testing each component of $\beta(\omega_0)$. Since successive testing is employed we need to apply the Bonferroni correction to the level of significance[7].

[7] The Bonferroni correction proceeds by testing each individual hypothesis at a significance level given by $\left(\frac{\alpha}{m}\right)$, where α is the desired level of significance and *m* is the number of hypotheses (see Mittelhammer et al (2000), Shaffer (1995) etc.).

## 5. An Application To The Decoupling Problem In Economics

Opening up of several emerging market economies in the last four decades or so, has led to remarkable rise in global linkages on trade, investment, technology and finances. There has been increasing debate in recent years on the hypothesis that the growth in Asian economies in general and the BRICS[8] countries, in particular, may now be progressively independent from the developments in the advanced economies ( see Willett et al (2011) for a history of this phenomenon – referred to as *decoupling*). The hypothesis did receive a setback in the aftermath of the Asian crisis (1997-98), but the quick recovery of the East Asian region from the crisis as well as the impressive growth in the BRICS group of countries in the decade following the Asian Crisis – a growth largely unaffected by the slowdown and short-term recession in the U.S. in the wake of the dot.com bust of 2000-01 -- once again provided a fresh impetus to it (see e.g. Akin and Kose (2008), Kose et al (2008), Fidrmuc et al (2008) etc). The recent global financial crisis in the US and Europe initially put the decoupling hypothesis under some strain, but the hypothesis seems to have received renewed attention , as the major emerging market economies have already resumed their pre-crisis robust growth trajectories, even though significant global recovery is still unavailable (see Nachane and Dubey (2013) and Pesce (2016) for a recent survey).

*Decoupling* is not a precisely defined term, and one needs to cautiously define the specific meaning in which the term is being used before applying it in any empirical context.[9] Some authors (e.g. Rossi (2009)) define decoupling as “growth in one area becoming less dependent on growth in another area”. Others seem to be defining it in terms of size of the growth spillovers from one economy to another (see IMF (2007)). Still others think of *decoupling* as delinking of EMEs from business cycles in the advanced countries. It is in this last sense that we will be interpreting the *decoupling* phenomenon in this paper. Our view therefore, corresponds in many senses, to the view that business cycles in EMEs are delinked (or de-synchronized) from cyclical movements in the advanced economies. *Decoupling*, in this sense, should not be seen as complete independence of EMEs from the adverse spillovers of shocks originating in the

[8] The term BRICS is an acronym used for the following group of countries—Brazil, Russia, India, China and South Africa.

[9] Willett et al (2011) offer a detailed discussion of the various senses in which the term *decoupling* has been used in the literature.

developed world – rather, it has a more modest subtext, implying a gradually decreasing reliance or dependence of a few large EMEs on the medium-term macroeconomic evolution of the advanced economies.

## 6. Empirical Results

Decoupling phenomenon in the literature has been analyzed via different methods and vary widely in scope and coverage. Akin and Kose (2008) studying a data set consisting over a hundred countries, use a *dynamic factor* model to uncover several interesting patterns in international linkages. Other studies have been limited in scope focusing on the dynamic nature of associations in different regions or groups of countries[10], with the output or equity market correlations being the preferred variable for analysis.

In this paper we propose to investigate the decoupling hypothesis using the spectral envelope approach for the following groups of countries[11] (i) the advanced group of developed economies comprising the EU (considered as a whole) and the US (ii) the BRICS group of countries (see Footnote 8 above for its composition) and (iii) as a representative of middle income but fast growing economies, we select Indonesia. The total number of countries thus is 8 (with the EU considered as a single unit). As the spectral envelope method is data-intensive, we need to use a measure of output which is available at a considerably high frequency. Since, for most EMEs, GDP data is available only at annual (or in rare cases quarterly) frequency, we have to settle for the Index of Industrial Production (IIP) on which data is available on monthly frequency. The limitation of this choice, however stems from the fact that the increasing part of the GDP in EMEs is now being accounted for by the services sector.

Our data span is from January 2000 to March 2017 yielding a total of $n = 207$ observations. The data is sourced from *OECD.Stat* (https://stats.oecd.org/index.aspx?queryid=207#).

[10] Fidrmuc et al (2008) focus on China, Jayaraman et al (2009) and Nachane and Dubey (2018) focus on India and Hsiao et al (2003) study the impact of the U.S. economy on the Asia-Pacific Rim.

[11] Three considerations dominated our choice of countries. Firstly, the data for the chosen country (on income or its proxy like Index of Industrial Production) should be available for a sufficient length of time and at a sufficiently high frequency. Secondly, the data should be at least broadly comparable across countries (and preferably available from a common database). Thirdly, the number of countries selected cannot be too large as spectral methods in general and the spectral envelope results in particular are heavy consumers of degrees of freedom with almost all results holding only asymptotically.

The results of our analysis are reported in Tables 1 to 4. As Table 1 indicates, 9 significant peaks in the spectral envelope, were identified via the criterion (11). Of these 9 peaks, only the first three viz. $\omega_1, \omega_2$ $and$ $\omega_3$ can claim economic relevance, the remaining being all very close to the Nyquist frequency. We therefore confine our attention only to the first three peaks, of which $\omega_1$ corresponding to a period of approximately 42 months (three and a half years), could indicate the possibility of a common medium term business cycle among the 8 countries considered. The other two peaks considered viz. $\omega_2$ $and$ $\omega_3$ correspond to cycles ranging from 7 to 8 months, possibly indicating a very short term business cycle or a seasonal component in common.

We next examine how many of the countries considered share common periodicities at the three peaks singled out for attention. For a particular peak (say $\omega_1$), we test each individual component of the corresponding eigenvector (say $\beta_8$ for $\omega_1$) for significance using the statistic $\psi$ defined in (15). A significant value of $\psi$ for that particular component points to the component being significant. Since each component is identified with a particular country ($\beta_8$ for $\omega_1$, is identified with Indonesia), this result can be interpreted as the corresponding country (Indonesia in our particular case) having a pronounced periodicity at that frequency. The analysis can be repeated for each country, by testing the corresponding eigenvector component for significance. The results of our analysis for the three frequencies $\omega_1, \omega_2$ $and$ $\omega_3$ are reported in Tables 2, 3 and 4 respectively.

The main result to emerge robustly from our analysis is that all the countries in our sample have a common cycle of approximately 42 months (see Table 2). Since our sample is fairly representative including the U.S., EU (developed world), BRICS (dominant EMEs) as also representative of the newly growing Asian economies (Indonesia), the results may be said to point to the possibility of a common global cycle between 3 to 4 years. Our results for the higher frequency cycles need more cautious interpretation for two reasons. Firstly, because the frequencies are very close to the Nyquist frequency. Secondly, in spite of the periodicities being in close proximity, the results differ somewhat – at frequency $\omega_2(= 0.126)$ all the countries seem to share a common cycle, whereas at the slightly higher frequency $\omega_3 = (0.140)$, India and South Africa stand out from the other countries.

## 7. Conclusions

In this paper, we have attempted to introduce the powerful recent technique of the spectral envelope into the analysis of an important problem in global economics. Based on this analysis we have been successful in isolating periodicities at which economic cycles across several countries are synchronized. In effect our results point to a strong possibility of a global business cycle of a periodicity between 3 to 4 years. As such the analysis indicates the potential for the applicability of the technique to economic series, where periodic features are the norm rather than the exception.

Some cautionary notes have to be introduced so that the results are not extrapolated beyond their legitimate jurisdiction. Firstly, the tests presented are all asymptotic and their small sample properties are unknown. Secondly, it should not be forgotten that (as seen in Section 2 above), the class of transformations $\mathcal{H}$ is restricted to be linear. Perhaps a richer structure might be unveiled by considering some polynomial approximations (Legendre, Hermite etc.) to a non-linear class of transformations. Finally, it would be useful if the concept of the spectral envelope were extended to the *time-frequency* context based on concepts such as the short-time Fourier transform (see Allen (1977), Naidu (1995, p. 222), evolutionary spectrum (see Priestley (1981), Wigner-Ville distribution (see Cohen (1989)) etc.

An important limitation of our study from the viewpoint of economic policy, is that we have not delved into the determinants of business cycle synchronization across countries. Undoubtedly global trade and financial integration has strong implications for the synchronization phenomenon, though the precise nature of these implications has been the subject of much controversy. If, as shown by Eichengreen (1992) and Krugman (1993), trade linkages are of the traditional *inter-industry* variety, then trade and financial linkages will lead to greater specialization of production. This, in turn, would lead to decreased business cycles synchronization, especially in the presence of industry-specific technological shocks. Frankel and Rose (1998), however, mention the possibility that, if the trade linkages were of the *intra-industry* (rather than *inter-industry*) variety, then the business cycles will be closely synchronized. Shin and Wang (2003) draw attention to the further possibility that increased trade linkages by creating a greater need for international coordination of fiscal and monetary policies, could homogenize policy responses to shocks and in the process lead to a greater degree of cyclical synchronization. A similar ambiguity is apparent with regard to financial linkages. Freer financial flows, by allocating capital in consonance with a country's production advantages, could reduce output co-movements. However, integration of global financial markets may also tighten the business cycle synchronization via wealth effects. We could not consider such additional factors affecting decoupling, mainly due to the non-availability of several key variables at a frequency sufficiently higher to implement the methods discussed.

Notwithstanding these limitations, it is hoped that the spectral envelope approach offers a promising new avenue for data exploration in economics.

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TABLE 1: Test of significance for Spectral Envelope Peaks

| (1) | (2) | (3) |
|---|---|---|
| Spectral Envelope Peaks at Frequencies (ω) | Amplitude of Spectral Envelope λ(ω) | Test Statistic for Significance of Peak (Right-hand side of (11)) |
| $\omega_1 = 0.024$ | 45.850** | 0.0228 |
| $\omega_2 = 0.126$ | 1.083** | 0.0228 |
| $\omega_3 = 0.140$ | 1.002** | 0.0228 |
| 0.208 | 1.205** | 0.0228 |
| 0.304 | 1.2848** | 0.0228 |
| 0.343 | 0.759** | 0.0228 |
| 0.396 | 1.014** | 0.0228 |
| 0.430 | 1.079** | 0.0228 |
| 0.464 | 1.220** | 0.0228 |

**Note:** 1.The desired level of significance is α=0.05, but since we are testing 9 peaks in succession the level of significance used is (by Bonferroni Correction) 0.0056.
2. (*) and (**) denote significance at 0.0056% and 0.001% respectively.

TABLE 2: Test Results ($\omega_1 = 0.024$)

| Country | $\beta(\omega_1)$ | $\psi$ (Test statistic for null hypothesis $\beta_j = 0\,; j = 1 \ldots 8$) |
|---|---|---|
| EU | 0.4539 | 32134.94** |
| US | 0.4452 | 47225.66** |
| Russia | 0.4164 | 27627.38** |
| South Africa | 0.3726 | 16290.99** |
| Brazil | 0.3592 | 28785.10** |
| India | 0.3332 | 41613.29** |
| China | 0.1899 | 13302.96** |
| Indonesia | 0.0845 | 2280.64** |

**Note:** 1. The statistic $\psi$ is as defined in (14) and the null hypothesis is not rejected if $\chi_6^2(0.003) \leq \psi \leq \chi_7^2(0.997)$ where $\chi_m^2(\alpha)$ is the ordinate which marks off an area of α% to the left of a $\chi_m^2$ (see (15) of the text). The level of significance used is 0.006% (using the Bonferroni correction).
2. (*) and (**) denote significance at 0.006% and 0.001% respectively.

TABLE 3: Test Results ($\omega_2 = 0.126$)

| Country | $\beta(\omega_2)$ | $\psi$ (Test statistic for null hypothesis $\beta_j = 0\,; j = 1 \dots 8$) |
|---|---|---|
| China | 0.6323 | 149263.94** |
| Russia | -0.4459 | 10512.34** |
| India | -0.4190 | 5740.51** |
| Brazil | -0.3620 | 16709.41** |
| EU | -0.1980 | 1269.61** |
| Indonesia | 0.1923 | 2791.82** |
| South Africa | 0.1017 | 296.49** |
| US | 0.0896 | 353.1167** |

Note: Same as in Table 2.

TABLE 4: Test Results ($\omega_3 = 0.140$)

| Country | $\beta(\omega_3)$ | $\psi$ (Test statistic for null hypothesis $\beta_j = 0\,; j = 1 \dots 8$) |
|---|---|---|
| China | 0.7532 | 93427.58** |
| Indonesia | -0.6152 | 38609.45** |
| US | 0.1360 | 679.29** |
| Brazil | -0.1268 | 3341.66** |
| EU | -0.1122 | 452.32** |
| Russia | 0.0803 | 3348.52** |
| South Africa | -0.0207 | 7.51 |
| India | -0.0012 | 0.05 |

Note: Same as in Table 2.