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Corruption vs. Efficiency in Water Allocation under Uncertainty: Is There a Trade-off?

Rupayan Pal and Dipti Ranjan Pati

Indira Gandhi Institute of Development Research, Mumbai
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Email(corresponding author): rupayan@igidr.ac.in

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Keywords: domestic water conflict, allocation rule, corruption, efficiency, third party adjudication

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Corresponding Author and Address:

Rupayan Pal, Indira Gandhi Institute of Development Research (IGIDR), Film City Road, Goregaon (E), Mumbai – 400065, India. Phone: +91-22-28416545.

Emails: (Pal) rupayan@igidr.ac.in, rupayanpal@gmail.com; (Pati) dipti@igidr.ac.in, searchme.dipti@gmail.com.

1. Introduction

Shared water, almost everywhere, remains a source of conflict, which becomes notoriously intractable under arid conditions. It is well argued that, under conditions of pure conflict, a negotiated solution is not possible as the initial allocation of rights itself is at stake (Richards and Singh, 2002). In the absence of negotiation between disputing parties, third party intervention seems to be the most immediate way out, particularly in case of within-country water disputes.[1] Allocation of water by a central authority remains a frequently used means of assigning disputed water, especially in developing countries that face difficulties in allowing the free market to take over. This is because high transaction costs and the lack of contract enforcement obstruct the bargaining process over water (Richards and Singh, 2001). Further, for water to be efficiently allocated by the free market, there has to be a system of pure private property rights, which is largely absent from the character of any of the water doctrines followed around the world, namely, riparian rights, public allocation and prior allocation (Sampath, 1992). In the absence of any possibility of a negotiated solution or the institutional framework necessary for water markets, an enforceable public allocation system appears to be the most plausible way of water sharing under pure conflict within a country.

In this paper, we analyse the issue of water sharing between an upstream region and its downstream counterpart in a federal arrangement considering that dispute resolution by a central planner is binding upon the parties involved. The legal provisions in various federations show that the necessary institutional framework exists to render binding any centralized third party intervention in case of interstate conflicts. For example, there are three ways of resolving interstate conflicts in the USA – by congressional act, by the formation of an interstate compact approved by the congress, or by an 'equitable apportionment' by the Supreme Court (Bennett, Howe and Shope, 2000). The constitution of India, via entry 56 in the Union List and article 262, enables the federal government to legislate and intervene effectively in case of interstate disputes and gives it primacy over the Supreme Court, even though water issues fall within the jurisdiction of the states as in the USA (Richards and Singh, 2002). The Interstate River Water Disputes Act of 1956 in India

[1] We note here that, in case of trans-boundary water flows, third party interventions may not be very effective due to lack of enforceability and it may also lead to inefficiency by jeopardizing the possibilities of cooperation (Ansink and Weikard, 2009).

provides for the establishment of tribunals by the central government in the event of negotiations failing. Such tribunals are often considered to be an extension of the federal government and are not beyond influence, political or otherwise, and corruption (Katz and Moore, 2011; Richards and Singh, 2002). However, to the best of our knowledge, the issue of influence and corruption has not received much attention in the existing theoretical literature on water disputes. This paper makes a modest attempt to fill this gap.[2]

This paper considers a situation in which water flow in a river is uncertain and two regions, an upstream region and a downstream region, of a country have dispute concerning water rights. The conflict between the two regions is defined over the entire range of water flow, so that flooding is as problematic as scarcity. The demand for water could be driven by a host of exogenous factors like historical use or complementary investment in infrastructure. To keep the analysis focused, it is assumed that ancillary investments have already been undertaken, determining the contours of the conflict ex-ante. The dispute over water sharing between the two regions is resolved through intervention by a third party (henceforth, the central planner or planner).

The central planner is considered to be corruptible and both upstream and downstream regions try to influence the planner's choice of water allocation through contributions. The contributions can be thought of as encompassing a gamut of political influence-building tactics (monetary or otherwise) undertaken by the disputing regions. The central planner favours one region over the other region, if the former contributes more than the latter; otherwise, the central planner’s water allocation decision is free from any bias. Each region is interested in maximizing its own benefit from water usage net of its cost of contribution, while the central planner’s objective is to maximize total contribution. It is assumed that preferences of each of the three players, i.e. of the central planner and the two disputing regions, and the implication of contributions by region(s) are exogenously determined and are common knowledge.

[2] It is observed that parties involved in disputes over water rights do not comply with the water sharing agreements in some cases, despite the agreement being legally binding. Bennett and Howe (1998) examine factors behind non-compliance in interstate river compacts. Ambec, Dinar and McKinney (2013) explore conditions for water sharing agreements to be sustainable to reduced flows. Richards and Singh (2002) argue that noncompliance might stem from a perverse combination of political affiliations that is not compatible with the states’ incentives and centre-state politics play crucial role in this regard. This paper, however, sidesteps the issue of compliance for simplicity.

This paper considers a three stage sequential move game. In the first stage of the game, the central planner chooses one of the two most frequently used water allocation rules – fixed allocation or proportional allocation. Next, in the second stage, the upstream region and the downstream region simultaneously and independently decide their respective contribution levels. Finally, in the third stage, the central planner decides the fixed amount or the proportion of total water flow to be allocated to the downstream region depending on the allocation rule decided in the first stage, before uncertainty regarding total water flow is resolved. Considering the sub-game perfect Nash equilibrium water allocation of this game to be legally binding and enforceable, this paper demonstrates the following.

It shows that the proportional allocation rule leads to higher total benefits from water usage by the two regions and, thus, is more efficient than the fixed allocation rule, under uncertainty. A benevolent central planer always chooses the efficient allocation rule. Interestingly, corruption does not necessarily lead to inefficiency. The corrupt central planner chooses the more (less) efficient allocation rule, i.e. proportional (fixed) allocation rule, if at the average level of water flow the problem of severe water scarcity does not (does) occur. This is because each region contributes more in the equilibrium under proportional allocation rule compared to that under fixed allocation rule, if expected flow of water is sufficiently large; otherwise, the opposite occurs. It implies that whether efficiency will be compromised in case of corruption or not depends on the state of nature. However, note that the corrupt central planner never aims to achieve efficiency, it occurs in some cases as mere coincidence.

We note here that this paper is closely related to Bennett, Howe and Shope (2000). Considering fixed and proportional water allocation rules, they compare efficiency of interstate water compacts based on two alternative allocation rules and evaluate their sensitivities to mean water flow and variance. While they also allow for the central planner to be biased towards one of the two regions, in their model the extent of bias is exogenously determined. On the contrary, in this paper, possible bias of the central planner is endogenously determined. Further, unlike Bennett, Howe and Shope (2000), this paper assesses implications of alternative allocation rules on corruption and the effect of corruption on equilibrium allocation.

This paper presents a simple model of behaviour often found in the actions of agents bound up in a tangle of political necessities and reciprocities. The planner here is as much a rational agent as the rest of the players and so, it is futile to expect him not to take care of his own interests. This sort of scenario fits quite well into the landscape of federal politics where the central government invariably seeks out support from lower levels of government and in return, bestows upon them favourable verdicts. The eventual nature of alignments or coalitions is determined through nothing else but the calculus of relative benefits drawn from various partners.

The rest of the paper is organized as follows. Section 2 presents the model, characterizes equilibrium outcomes under alternative rules of water allocation (Section 2.1 and Section 2.2), and analyses the corrupt planner's optimal choice of the rule (Section 2.3). Analysis of the efficient allocation rule and its comparison with the corrupt planner's optimal choice is presented in Section 3. Section 4 concludes. Proofs are presented in the Appendix.

2. The Setup

There are two regions, upstream (U) and downstream (L), with conflicting interests over a shared river. These two regions and the river are located within the boundary of a single country, which has a federal setup. We mention here that regions in this model could be interpreted as firms or any productive entities with claims to a shared river. Water flow, W (≥ 0), in the river is random and is assumed to have the following distribution

$$W = \begin{cases} W_h \; with\; probability\; \rho \in (o, 1) \\ W_l \; with\; probabiltiy\; 1 - \rho \end{cases} \quad (1)$$

Region i's benefit from water usage is given by $B_i(W_i)$, where $W_i(\geq 0)$ denotes the amount of water used by region i; $B_i(0) = 0$, $B_i{}'(W_i) \gtreqless 0$ if $W_i \lesseqgtr W_i{}^*$, and $B_i{}''(W_i) < 0 \;\; \forall \, W_i \geq 0$; $i = U$, L. Johnson, Gisser and Werner (1981) argue that for efficient allocation of water, property rights must be defined in terms of consumptive use and not diversion. Following this argument, we assume that the entire water allocation is used for consumptive purposes.

Since $B_i(W_i)$ is strictly concave in W_i, it has a unique maximum. Let $B_i(W_i)$ attains its maximum at $W_i = W_i{}^*$, $i = U,\; L$. Now, if $W_l < W_i{}^* < W_h,\; i = U,\; D,$ the probability of the event of water scarcity and the probability of the event of flood are both strictly positive for each region in case that region receives the entire water flow (W). Further note that, if $W < W_U^* + W_L^*$, there is the problem of scarcity. Alternatively, if total water flow $W > W_U^* + W_L^*$, there is the problem of flooding.

We assume (a) $W_l < W_i^* < E(W),\ i = U,\ L$ and (b) $W_U^* + W_L^* < W_h$, which implies that each of the two situations – water scarcity and flooding – occurs with positive probability, regardless of whether water is shared between the two regions or only one region receives the entire flow. In either of the two situations, scarcity or flooding, there are disputes between the two regions over water sharing, which they fail to resolve by themselves through cooperation. The central planner, who is a third party, intervenes in the process of dispute settlement and resolves the dispute for once and for all. Allocation of water prescribed by the central planner is considered to be legally binding and enforceable.

In order to resolve the dispute, the central planner first chooses the allocation rule, which can be either a fixed allocation rule or a proportional allocation rule. In case of fixed allocation rule, the central planner chooses a fixed amount of water $W_0 \in (0,\ W_h)$ such that the downstream region will receive (a) W_0 amount of water, if $W_0 < W$, or (b) the entire water W, if $W_0 \geq W$, and the upstream region will receive the remaining water, if any. On the other hand, in case of proportional allocation rule, the central planner decides the proportion $\beta \in (0,\ 1)$ of total water flow W to be allocated to the downstream region, which implies that the upstream region will receive $(1 - \beta)W$ amount of water. The central planner chooses $W_0 \in (0,\ W_h)$ or $\beta \in (0,\ 1)$, depending on the predefined allocation rule, so that the expected value of a weighted sum of regions' benefits from water usage is maximum, which is as follows.

$$E[Z] = E[B_U(W_U) + \lambda B_L(W_L)], \tag{2}$$

where $\lambda (\geq 0)$ is the weight assigned by the central planner to the downstream region's benefit *a la* Bennett, Howe and Shope (2000). The weight parameter λ measures the bias of the central planner. The central planner is said to be biased towards the downstream (upstream) region, if $\lambda > 1$ ($\lambda < 1$). However, unlike Bennett, Howe and Shope (2000), this paper endogenously determines the central planner's bias λ. Let us consider that the central planner's relative bias depends on the contributions made by both regions. It is reasonable to assume that if the downstream region's contribution $C_L \in [0,1]$ is greater than the upstream region's contribution $C_U \in [0,1]$, i.e. if $C_L > C_U$, the central planner will be biased towards the downstream region ($\lambda > 1$); while the opposite occurs, if $C_L < C_U$. It is assumed that the central player's bias is given by $\lambda = \lambda\,(C_L, C_U)$, where $\lambda\,(0,0) = 1$, $\frac{\partial\lambda}{\partial C_U} < 0$, $\frac{\partial\lambda}{\partial C_L} > 0$. Both $\lambda\,(C_L, C_U)$ and Z are assumed to be common knowledge.[3]

Each region incurs cost to make contribution. Let region i's cost to contribute C_i be given by $\frac{\psi {C_i}^2}{2}$, where $\psi > 0$ is the cost parameter; $i = U,\ L$.

There are three stages of the game as follows.

Stage 1: The central planner chooses the water allocation rule, fixed allocation vs. proportional allocation, which maximizes the sum of the contributions from the two regions $O = C_L + C_U$.

Stage 2: The two regions decide their respective levels of contribution they make to the planner, simultaneously and independently.

Stage 3: The central planner decides the fixed amount (W_0) or the proportion (β), depending on the allocation rule decided in the first stage, of water to be allocated to the downstream region, such that the expected weighted benefit $E[Z]$ is maximized.

We solve this game by backward induction by considering that $B_i(W_i) = aW_i - \frac{b}{2}\,{W_i}^2$ and $\lambda = \lambda\,(C_L, C_U) = 1 + C_L - C_U$, for simplicity, where $a\ (> 0)$ and $b\ (> 0)$ are benefit parameters; $i = U,\ L$. We assume that $\psi \geq Max[\frac{(2a - b\overline{W})^2}{8b}, \frac{(2a\overline{W} - bE(W^2))^2}{8bE(W^2)}]$ and $\overline{W} > \frac{a}{b}$, which ensures existence of unique interior equilibrium and stability of the equilibrium in each of the two cases, fixed and proportional, considered in this paper. We first solve stage 3 and then stage 2, by considering fixed allocation rule and proportional allocation rule separately.

[3] The central planner's bias may be his private information and different regions may have different beliefs about it. In case each region has the same belief regarding the form of the bias function $\lambda\,(C_L, C_U)$, the qualitative results of this paper go through.

2.1 Fixed Allocation Rule

Let us first consider that fixed allocation rule has been chosen by the central planner in the first stage of the game. In this case, the central planner decides the fixed amount of water $W_o \in (0, W_h)$ in stage 3, which implies that water flows in the downstream region and in the upstream region are, respectively, as follows.

$$W_L = \begin{cases} W_0, & if \;\; W_0 < W \\ W, & if \;\; W_0 \geq W \end{cases} \quad \text{and} \quad W_U = \begin{cases} W - W_0, & if \;\; W_0 < W \\ 0, & if \;\; W_0 \geq W \end{cases} \tag{3}$$

Note that, since $W_o \in (0,\ W_h)$, $Prob(W_0 < W) = \nu = \begin{cases} 1, & if \;\; W_0 < W_l \\ \rho, & if \;\; W_l \leq W_0 < W_h \end{cases}$ and $Prob(W_0 \geq W) = 1 - \nu$. Therefore, expected weighted benefits from water usage by the two regions under fixed allocation rule is as follows, where subscript F indicates fixed allocation rule.

$$E[Z]_F = E[B_U(W_U) + \lambda\, B_L(W_L)]_F$$

$$= \nu E\left[\left\{a(W - W_o) - \frac{b}{2}(W - W_o)^2\right\} + \lambda\left\{aW_o - \frac{b}{2}\, W_o{}^2\right\}\right] + (1-\nu)E[\lambda\left\{aW - \frac{b}{2}\, W^2\right\}] \tag{4}$$

Solving the problem of the central planner in stage 3, $\max\limits_{W_o \in (0, W_h)} E[Z]_F$, we obtain the following.

Lemma 1: *Suppose that the fixed allocation rule is in force. Then, for any given bias of the central planner* $\lambda\ (\geq 0)$, *the central planner's optimal choice of the fixed amount of water is given by* $W_o(\lambda) = \frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)} \in (0, W_h)$ *and the corresponding equilibrium gross benefits from water usage of the downstream region and the upstream region are, respectively, as follows.*

$$E[B_L(W)\,|W_o = W_o(\lambda)] = \nu\, B_L(W_o(\lambda)) + (1-\nu)E[B_L(W)],\ \textit{and}$$

$$E[B_U(W)\,|W_o = W_o(\lambda)] = \nu\, EB_U\big(W - W_o(\lambda)\big)],\ \textit{where}\ \nu = \begin{cases} 1, & if \;\; W_o(\lambda) < W_l \\ \rho, & if \;\; W_l \leq W_0(\lambda) < W_h \end{cases}.$$

Proof: See Appendix.

It is easy to check that

$$\frac{\partial W_o(\lambda)}{\partial \lambda} = -\frac{\partial\big(\overline{\mathrm{W}} - \mathrm{W_o}(\lambda)\big)}{\partial \lambda} = \frac{2a - b\overline{W}}{b(1+\lambda)^2} > (<) 0, \text{ if } \overline{W} < (>) \frac{2a}{b}; \tag{5a}$$

$$\frac{\partial W_o(\lambda)}{\partial \overline{\mathrm{W}}} = \frac{1}{1+\lambda} > 0 \;\; and \;\; \frac{\partial(\overline{\mathrm{W}} - \mathrm{W_o}(\lambda))}{\partial \overline{\mathrm{W}}} = \frac{\lambda}{1+\lambda} \geq 0, \forall\, \lambda \geq 0 \tag{5b}$$

So, the fixed allocation to the downstream region is increasing in the weight λ, if $\bar{W} < \frac{2a}{b}$; while it decreases with λ, if $\bar{W} > \frac{2a}{b}$. Further, if $\bar{W} < \frac{2a}{b}$, $\mathrm{W_o} < \frac{a}{b}$ and $(\bar{\mathrm{W}} - \mathrm{W_o}) < \frac{a}{b}$. So, if the planner's bias towards the downstream region is more, he allocates more water to the downstream region in case water scarcity is expected ($\bar{W} < \frac{2a}{b}$), while he allocates less water to the downstream region in case flood is expected to occur ($\bar{W} > \frac{2a}{b}$). Further, from (5b) it follows that each region gets more water when the expected water flow is higher, but the positive effect of expected flow on downstream (upstream) region's share decreases (increases) with the increase in central planner's bias towards the downstream region. It implies that the central planner serves the interest of the downstream region (by tempering the award in a situation of overabundance), at the expense of the upstream region, when he is biased towards the downstream region.

Under fixed allocation rule, efficient fixed allocation ($W_0^{Efficient}$) maximizes the total benefit from water usage by the two regions and, thus, $\mathrm{W_o} = W_0^{Efficient}$ equates downstream region's expected marginal benefits from water usage to that of upstream region: $\frac{\partial E[B_L(W_o)]}{\partial W_o} = \frac{\partial E[B_U(W-W_o)]}{\partial (W-W_o)}$. In contrast, in the present scenario the central planner's optimal choice of $\mathrm{W_o}$ is given by $\lambda \frac{\partial E[B_L(W_o)]}{\partial W_o} = \frac{\partial E[B_U(W-W_o)]}{\partial (W-W_o)}$, i.e. at the central planner's optimal choice, the upstream region's expected marginal benefit is less (greater) than the downstream region's marginal benefit when the central planner is biased towards (against) the upstream region, i.e. when $\lambda < 1$ ($\lambda > 1$). It follows that, if the central planner is biased towards (against) the upstream region, its optimal choice of the fixed amount of water for the downstream region is less (more) than the efficient level.

Let $E[G_{i,F}]$ denote the expected net benefit from water usage of region i $(= U, L)$ under fixed allocation rule, i.e. $E[G_{i,F}] = E(B_i)_F - \frac{\psi C_i{}^2}{2}$, $i = U, L$; where $E(B_i)_F = E[B_i(W)\ |W_o = W_o(\lambda)]$. It is easy to observe the following.

$$\left.\frac{\partial E[G_{L,F}]}{\partial C_L}\right|_{C_L=0} = \left[\underbrace{\frac{\partial E(B_L)_F}{\partial \mathrm{W_o}}}_{(+/-)} \underbrace{\frac{\partial \mathrm{W_o}}{\partial \lambda}}_{(+/-)} \underbrace{\frac{\partial \lambda}{\partial C_L}}_{(+)}\right]_{C_L=0} > 0 \text{ and} \tag{6a}$$

$$\left.\frac{\partial E[G_{U,F}]}{\partial C_U}\right|_{C_U=0} = \left[\underbrace{\frac{\partial E(B_U)_F}{\partial(\bar{W} - \mathrm{W_o})}}_{(+/-)} \underbrace{\frac{\partial(\bar{W} - \mathrm{W_o})}{\partial \lambda}}_{(-/+)} \underbrace{\frac{\partial \lambda}{\partial C_U}}_{(-)}\right]_{C_U=0} > 0 \qquad (6b)$$

From (6a) and (6b), it follows that it is optimal for each of the two regions to make positive contributions to the central planner. The reason is as follows. Note that the central planner's bias towards the downstream region is increasing (decreasing) in downstream (upstream) region's contribution: $\frac{\partial \lambda}{\partial C_L} > 0$ and $\frac{\partial \lambda}{\partial C_U} < 0$, $\forall\, C_L, C_U \in [0, 1]$. Further, if mean of total water flow is less than a critical level, (a) the central planner allocates higher (lower) amount of water to the downstream (upstream) region in case he is more biased towards the downstream region and (b) a region's expected gross benefit from water usage is increasing in expected water flow in that region; otherwise, opposites are true.[4]

Lemma 2: *In the case of fixed allocation rule under uncertainty, the following hold.*

- *a) The downstream region perceives that its own contribution and its rival's contribution are strategic complements.*
- *b) The upstream region perceives that the two regions' contributions are strategic complements (substitutes), if $C_U > C_L + \frac{1}{2}$ $\;(C_U < C_L + \frac{1}{2})$.*

Proof: See Appendix.

Lemma 2 implies that it is always optimal for the downstream region to increases its contribution in response to an increase in the upstream region's contribution, i.e. the downstream region's contribution-reaction function is always upward sloping in the $C_L C_U$ plane. However, corresponding to an increase in the downstream region's contribution, the upstream region's best response is to reduce its contribution unless the downstream region's contribution is sufficiently low, as depicted in Figure 1.

[4] If $\bar{W} < \frac{2a}{b}$, we have $\frac{\partial \mathrm{W_o}}{\partial \lambda} > 0$, $\frac{\partial(\bar{W} - \mathrm{W_o})}{\partial \lambda} < 0$, $\frac{\partial E(B_L)_F}{\partial \mathrm{W_o}} > 0$ and $\frac{\partial E(B_U)_F}{\partial(\bar{W} - \mathrm{W_o})} > 0$, since $\bar{W} < \frac{2a}{b} \Rightarrow \mathrm{W_o} < \frac{a}{b}$ and (5a) holds. Otherwise, if $\bar{W} > \frac{2a}{b}$, we have $\frac{\partial \mathrm{W_o}}{\partial \lambda} < 0$, $\frac{\partial(\bar{W} - \mathrm{W_o})}{\partial \lambda} > 0$, $\frac{\partial E(B_L)_F}{\partial \mathrm{W_o}} < 0$ and $\frac{\partial E(B_U)_F}{\partial(\bar{W} - \mathrm{W_o})} < 0$.

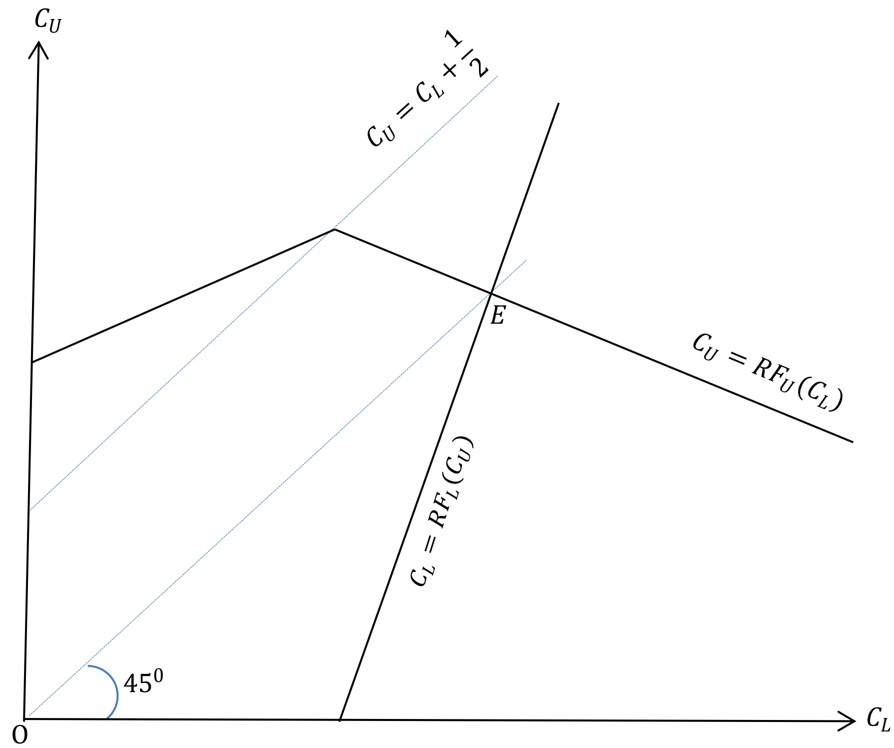


Figure 1: Contribution-Reaction-Functions (*RF*s) and the Equilibrium (*E*)

In stage 2, problems of region i is $\max_{C_i \in [0,1]} E[G_{i,F}]$, $i = U, L$. Solving these two problems simultaneously we obtain the following.

Lemma 3: *In the case of fixed allocation rule under uncertainty, the downstream region and the upstream region contribute equally in order to align with the central planner in the equilibrium and the equilibrium contribution of each region is equal to* $C_{fixed} = C_{U,F} = C_{L,F} = \nu \frac{(2a - b\overline{W})^2}{8b\psi} > 0$.

Proof: See Appendix.

Each region contributes a positive amount to influence the central planner's allocation decision in the equilibrium, although it incurs a sufficiently high cost in doing so. Further, since each region contributes the same amount, the central planner remains unbiased in the equilibrium: $\lambda_{fixed} = 1 + C_{L,F} - C_{U,F} = 1$. It implies that each region is worse off in the equilibrium when the central planner is corrupt compared to the scenario in which the central planner is not corruptible. Nonetheless, since contributions are transfers without leakage within the economy, total surplus remains the same, regardless of whether there is corruption or not, given that the fixed allocation rule is in force.

From Lemma 1 and Lemma 3, we obtain Lemma 4.

Lemma 4: *Under the fixed allocation rule, the sub-game perfect Nash equilibrium (SPNE) fixed amount chosen by the central planner* (W_0^*) *, expected water flow* $E[W_{i,F}]^*$ *to region* i $(= L, U)$ *and the corrupt central planner's payoff* O_F^* *are, respectively, as follows.*

(i) $W_0^* = \frac{\bar{W}}{2}$

(ii) $E[W_{L,F}]^* = \bar{W}(1 - \frac{\nu}{2})$ *and* $E[W_{U,F}]^* = \nu \frac{\bar{W}}{2}$.

(iii) $O_F^* = 2\, C_{fixed} = \nu \frac{(2a - b\bar{W})^2}{4b\psi}$.

2.2 Proportional Allocation Rule

We now turn to analyse the equilibrium under proportional allocation rule. When proportional allocation rule is in force, in stage 2 of the game the central planner decides the proportion, $\beta \in (0, 1)$, of water flow to be allocated to the downstream region. It implies that the downstream region will receive $W_L = \beta\, W$ amount of water, while the upstream region will receive $W_U = (1 - \beta)\, W$ amount of water, where W denotes the total flow of water in the river. Thus, expected benefits from water usage of the downstream region and the upstream region are, respectively, as follows.

$$E[B_L(\beta W)] = E\left[a\beta W - \frac{b}{2}\,(\beta W)^2\right] \tag{7a}$$

$$E[B_U\{(1-\beta)W\}] = E[a(1-\beta)W - \frac{b}{2}\{(1-\beta)W\}^2] \tag{7b}$$

Therefore, expected weighted benefits from water usage by the two regions under proportional allocation rule is given by

$$E[Z]_P = E[B_U\{(1-\beta)W\}] + \lambda\, E[B_L(\beta W)], \tag{8}$$

where $E[B_U\{(1-\beta)W\}]$ and $E[B_L(\beta W)]$ are given by (7b) and (7a), respectively, and subscript P denotes proportional allocation rule

Now, solving the central planner's problem in stage 3, $\max_{\beta \in (0,1)} E[Z]_P$, we get the equilibrium proportions of total water flow for the two regions, given their contributions, as in Lemma 5.

Lemma 5: *Suppose that proportional allocation rule is in place for water allocation. Then, given the central planner's bias (λ), in the equilibrium the proportion of water accruing to the downstream region is given by* $\beta^* = \beta(\lambda) = \frac{bE(W^2) - a(1-\lambda)\bar{W}}{b(1+\lambda)E(W^2)}$ *and that to the upstream region by* $(1 - \beta^*) = [1 - \beta(\lambda)] = \frac{a(1-\lambda)\bar{W} + b\lambda E(W^2)}{b(1+\lambda)E(W^2)}$, *whenever* $\beta(\lambda) \in (0,\ 1)$*; otherwise, (a) if* $\beta(\lambda) \geq 1$, $\beta^* = 1 - \epsilon$, *and (b) if* $\beta(\lambda) \leq 0$, $\beta^* = \epsilon$, *where* ϵ *is a small positive number.*

Proof: See Appendix

It is easy to check the following.

$$\frac{\partial \beta(\lambda)}{\partial \lambda} = -\frac{\partial(1-\beta(\lambda))}{\partial \lambda} = \frac{[2a\bar{W} - bE(W^2)]}{bE(W^2)(1+\lambda)^2} > 0 \Leftrightarrow \sigma^2 < \frac{\bar{W}}{b}\ [2a - b\bar{W}], \qquad (9a)$$

where $\sigma^2 = E[(W - \bar{W})^2] = E[W^2] - \bar{W}^2$.

$$\frac{\partial \beta(\lambda)}{\partial \bar{W}} = -\frac{\partial(1-\beta(\lambda))}{\partial \bar{W}} = \frac{a(1-\lambda)(\bar{W}^2 - \sigma^2)}{b(1+\lambda)[E(W^2)]^2} > 0 \Leftrightarrow \frac{\bar{W}}{\sigma} > 1 \text{and } \lambda < 1 \qquad (9b)$$

Assuming that $\beta(\lambda) \in (0,\ 1)$, from (9a) we can state the following. An increase in the central planner's bias towards the downstream region, i.e. an increase in λ, never leads to greater proportion of water accruing to the downstream region, if $\bar{W} \geq \frac{2a}{b}$ or σ^2 is greater than a critical level in case $\bar{W} < \frac{2a}{b}$, i.e. if there water abundance at the mean flow or if variation in water flow is sufficiently large. In other words, greater bias of the central planner towards the downstream region serves the downstream region's interests to a greater extent at the expense of the upstream region. Further, condition (9b) implies that the proportion of water allocated to the downstream region rises, while that to the upstream region falls, with increase in mean water flow when the coefficient of variation is higher than 1 and the planner cares less for the downstream region. When the mean water flow falls, ceteris paribus, the proportion allocated to the downstream region falls and that to the upstream rises. This essentially means that the planner, when he cares less about the downstream region, makes it bear a greater amount of risk in the face of highly variable water flow by reducing the downstream proportion with fall in mean flow.

Note that the central planner optimal choice of β is given by $\lambda \frac{\partial E[B_L(\beta W)]}{\partial \beta} = \frac{\partial E[B_U(W-\beta W)]}{\partial (1-\beta)}$; whereas efficient level of β (which maximizes joint benefit of water usage by the two regions) satisfies the condition of marginal benefit equalization across regions, $\frac{\partial E[B_L(\beta W)]}{\partial \beta} = \frac{\partial E[B_U(W-\beta W)]}{\partial (1-\beta)}$. Clearly, the corrupt central planner's (λ) optimal choice β^* is not efficient. If the central planner is biased towards the downstream region ($\lambda > 1$), the downstream (upstream) regions marginal benefit from water usage is over (under) emphasised by the central planner.

Let $E[G_{i,P}]$ denote the expected net benefit from water usage of region $i\,(= U, L)$ under proportional allocation rule, i.e. $E[G_{i,P}] = E(B_i)_P - \frac{\psi C_i^2}{2}$, $i = U, L$; where $E(B_i)_P = E[B_i(W)\,|\beta = \beta(\lambda)]$ (assuming interior solution). Then, we have the following.[5]

$$\left.\frac{\partial E[G_{L,P}]}{\partial C_L}\right|_{C_L=0} = \left[\underbrace{\frac{\partial E(B_L)_P}{\partial \beta}}_{(+/-)} \underbrace{\frac{\partial \beta}{\partial \lambda}}_{(+/-)} \underbrace{\frac{\partial \lambda}{\partial C_L}}_{(+)}\right]_{C_L=0} > 0 \qquad (10a)$$

$$\left.\frac{\partial E[G_{U,P}]}{\partial C_U}\right|_{C_U=0} = \left[\underbrace{\frac{\partial E(B_U)_P}{\partial (1-\beta)}}_{(+/-)} \underbrace{\frac{\partial (1-\beta)}{\partial \lambda}}_{(+/-)} \underbrace{\frac{\partial \lambda}{\partial C_U}}_{(-)}\right]_{C_U=0} > 0 \qquad (10b)$$

Clearly, from (10a) and (10b) it follows that (i) a region can induce the corrupt central planner to be biased in its favour by making contributions and (ii) each region would make positive contribution in the equilibrium under proportional allocation rule, as observed in the case of fixed allocation rule.

By examining how a region's marginal net benefit of its own contribution varies with its rival region's contribution, we obtain the following.

[5] It can be checked that either both $\frac{\partial E(B_L)_P}{\partial \beta}$ and $\frac{\partial \beta(\lambda)}{\partial \lambda}$ are positive, or both are negative. On the other hand $\frac{\partial E(B_U)_P}{\partial (1-\beta)}$ and $\frac{\partial (1-\beta)}{\partial \lambda}$ cannot be of the same sign.

Lemma 6: *In the case of proportional allocation rule under uncertainty, the following hold.*

a) *The downstream region perceives that the contributions of the two regions are strategic complements.*
b) *The upstream region perceives that the contributions are strategic complements (substitutes) if $C_U > C_L + \frac{1}{2}$ ($C_U < C_L + \frac{1}{2}$).*

Proof: See Appendix.

Interestingly, Lemma 2 and Lemma 6 together imply that regions' perceptions about strategic nature of their contributions do not depend on the rule of allocation, fixed or proportional. Thus, the implications of strategic natures of contributions as perceived by the two regions through their respective contribution-reaction functions and the equilibrium in the case of proportional allocation rule will be the same as in the case of fixed allocation rule. Now, solving the two regions' problems in stage 2, $\max_{C_i \in [0,1]} E[G_{i,P}]$, $i = U, L$, simultaneously, we derive the equilibrium contribution of each region under proportional allocation rule.

Lemma 7: *Suppose that the corrupt central planner allocates water between the two regions according to the proportional allocation rule. Then, it is optimal for each region to contribute the amount C_{pro} to influence the central planner's choice of proportions, where $C_{pro} = \frac{[2a\bar{W} - bE(W^2)]^2}{8b\psi E(W^2)}$.*

Proof: See Appendix

Lemma 7 states that each region contributes equally to the central planner in the equilibrium under proportional allocation rule, as under fixed allocation rule (Lemma 3). It follows that in the equilibrium, the central planner gives equal weight to each region's expected benefit from water usage ($\lambda^* = 1$) regardless of the rule of allocation.

From Lemma 5 and Lemma 7, we get the expected water flow in each region and the central planner's payoff in the equilibrium under proportional allocation rule.

Lemma 8: *Under the proportional allocation rule, the SPNE proportion of total water flow allocated to the downstream region (β^*), expected water flow $E[W_{i,P}]^*$ to region i $(= L, U)$ and the corrupt central planner's payoff O_P^* are, respectively, as follows.*

(i) $\beta^* = \frac{1}{2}$

(ii) $E[W_{L,P}]^* = E[W_{U,P}]^* = \frac{\overline{W}}{2}$

(iii) $O_P^* = 2C_{pro} = \frac{[2a\overline{W} - bE(W^2)]^2}{4b\psi E(W^2)}$

2.3 Corrupt Planner's Choice of Allocation Rule: Fixed vs. Proportional

Now the question is: what is the corrupt central planner's optimal choice of water allocation rule – fixed or proportional, in stage 1 of the game? Comparing the central planner's equilibrium payoff, which is given by total contribution received by the central planer from the two regions in the equilibrium, under the fixed allocation rule with that under the proportional allocation rule, we obtain the following.

Proposition 1: *In the sub-game perfect Nash equilibrium, the corrupt central planner's choice of water allocation rule under uncertainty is as follows.*

- *It is optimal for the corrupt central planner to choose the fixed allocation rule, if severe water scarcity is expected, i.e. if $\overline{W} \in (W_l, \underline{W}\ )$, where $\underline{W} \leq \sqrt{\frac{4a^2}{b^2} - \sigma^2}$.*
- *The corrupt social planner prefers the proportional allocation rule over the fixed allocation rule, if expected water flow in the river is either excessive ($\overline{W} > \frac{2a}{b}$) or optimal ($\overline{W} = \frac{2a}{b}$) or moderately scarce, i.e. $\overline{W} \in \left[\underline{W}, \frac{2a}{b}\right)$.*

Proof: See Appendix.

Note that in the absence of uncertainty in water flow, the proportional allocation rule is synonymous with the fixed allocation rule and, thus, regions contribute the same amount under alternative allocation rules. To illustrate it further, note that, whenever $\sigma = 0$, we must have $W_l =$

$W_h = \bar{W}$ and $O_P^* = O_F^* = \frac{[2a-b\bar{W}]^2}{4b\psi}$, by Lemma 4 and Lemma 8. Therefore, the corrupt social planner is indifferent between the two allocation rules in case water flow is certain, regardless of whether there is water scarcity or water abundance. This together with Proposition 1 implies that existence of uncertainty in water flow has a significant bearing on the corrupt social planer's equilibrium choice of the allocation rule.

3. Efficient Allocation Rule: Honest Social Planner

An allocation rule is said to be efficient, if that allocation rule results in maximum total expected benefit from water usage by the two regions. If the social planner is honest (i.e. not corruptible), he always strictly prefers the more efficient allocation rule. That is, the honest social planner will prefer the fixed allocation rule over the proportional allocation rule, if the sum of expected benefits from water usage, $E[B_U(W_U)] + E[B_L(W_L)]$, is greater under fixed allocation rule compared to that under proportional allocation rule. In such a scenario, the possibility of influencing the social planner's decisions through contributions does not exist, and thus $\lambda = 1$ holds true always.

Now, under the fixed allocation rule, the honest social planner chooses the fixed amount W_o by solving the following problem

$$\max_{W_o \in (0, W_h)} E[Z]_{F,\lambda=1} = \nu E\left[\left\{a(W - W_o) - \frac{b}{2}(W - W_o)^2\right\} + \left\{aW_o - \frac{b}{2}\, {W_o}^2\right\}\right]$$
$$+(1-\nu)E[0 + \left\{aW - \frac{b}{2}\, W^2\right\}]$$

Solving the above problem, we get $W_o = \frac{\bar{W}}{2} = W_o{}^{**}$. It implies that

$$E[Z]_{F,\lambda=1}{}^{**} = \nu\left[\frac{a\bar{W}}{2} + \frac{3b\bar{W}^2}{8} - \frac{bE(W^2)}{2}\right] + \nu\left[\frac{a\bar{W}}{2} - \frac{b\bar{W}^2}{8}\right] + (1-\nu)\left[a\bar{W} - \frac{bE(W^2)}{2}\right]$$
$$= a\bar{W} - \frac{bE(W^2)}{2} + \nu\frac{b\bar{W}^2}{4}$$

On the other hand, under proportional allocation rule, the honest social planner chooses the proportion β by solving the following problem.

$$\max_{\beta\in(0,1)} \mathrm{E}[\mathrm{Z}]_{\mathrm{P},\lambda=1} = \mathrm{E}[\mathrm{a}(1-\beta)\mathrm{W} - \tfrac{\mathrm{b}}{2}\{(1-\beta)\mathrm{W}\}^2] + \mathrm{E}\left[\mathrm{a}\beta\mathrm{W} - \tfrac{\mathrm{b}}{2}(\beta\mathrm{W})^2\right].$$

Solving the above problem we get $\beta = \frac{1}{2} = \beta^{**}$. Thus,

$$\mathrm{E}[\mathrm{Z}]_{\mathrm{P},\lambda=1}{}^{**} = \left[\frac{\mathrm{a}\overline{\mathrm{W}}}{2} - \frac{\mathrm{bE}(\mathrm{W}^2)\lambda^2}{8}\right] + \left[\frac{\mathrm{a}\overline{\mathrm{W}}}{2} - \frac{\mathrm{bE}(\mathrm{W}^2)}{8}\right] = \mathrm{a}\overline{\mathrm{W}} - \frac{\mathrm{bE}(\mathrm{W}^2)}{4}.$$

Note that

$$\mathrm{E}[\mathrm{Z}]_{\mathrm{P},\lambda=1}{}^{**} - \mathrm{E}[\mathrm{Z}]_{\mathrm{F},\lambda=1}{}^{**} = \frac{\mathrm{bE}(\mathrm{W}^2)}{4} - \nu\frac{\mathrm{b}\overline{\mathrm{W}}^2}{4} = \frac{\mathrm{b}}{4}[\overline{\mathrm{W}}^2(1-\nu) + \sigma^2] > 0.$$

Proposition 2: *The honest social planner always chooses the proportional allocation rule to allocate water under uncertainty, i.e. the proportional allocation rule is more efficient than the fixed allocation rule.*

Proof: Follows directly from the above discussion.

The intuition behind Proposition 2 is as follows. Note that $\lim_{\sigma\to 0}\nu = \lim_{\sigma\to 0} Prob(W > W_0) = 1$ and thus $\lim_{\sigma\to 0}\left(E[Z]_{P,\boldsymbol{\lambda}=\mathbf{1}}{}^{**} - E[Z]_{F,\boldsymbol{\lambda}=\mathbf{1}}{}^{**}\right) = 0$. It implies that in absence of uncertainty, the fixed allocation rule and the proportional allocation rule are equally efficient. Under uncertainty, the two disputing regions equally share the risk involved in the case of proportional allocation rule, whereas one of the two regions (the upstream region in the present analysis) bears disproportionately greater share of the risk in the case of fixed allocation rule. Since regions' benefit functions are considered to be the same, risk sharing is more efficient in the case of proportional allocation rule compared to that in the case of fixed allocation rule. Thus, when there is uncertainty in water flow, the proportional allocation rule turns out to be more efficient than the fixed allocation rule.

From Proposition 1 and Proposition 2, the following result is immediate.

Proposition 3: *When there is uncertainty in water flow, the corrupt social planner chooses the inefficient rule of water allocation only if severe water scarcity is expected to occur ($W_l < \bar{W} < \underline{W}$); otherwise, the SPNE under corruption is efficient.*

It is interesting to observe that while both water scarcity and excessive water flow are undesirable to each of the two regions, corruption leads to inefficiency in allocation only if the problem of severe water scarcity occurs at the average water flow.

4. Conclusion

In this paper we have developed a simple model to understand the implications of corruption on efficiency in water allocation between two conflicting regions by a third party (central planner) in a federal arrangement. Considering that the conflicting regions are symmetric, we have shown that, given the rule of water allocation – fixed or proportional, even under uncertainty the corrupt planner opts for efficient allocation of water between conflicting regions in the equilibrium. However, corruption has significant distortionary effects on the planner's choice of the rule of allocation when water flow is uncertain, unlike as in the case of certain water flow. In the equilibrium, the corrupt planner opts for inefficient rule of allocation in a scenario in which severe water scarcity occurs on an average. In other words, a corrupt planner compromises with efficiency for his own private benefit when severe water scarcity is expected to occur. In broader terms, this result seems to suggest that while corruption may not portend inefficiency in resource rich nations, prevalence of corruption does hurt efficiency of resource poor nations.

In this analysis we have assumed that conflicting regions are symmetric, which helps to clearly identify the effects of uncertainty in water flow and corruption. Intuitively we can say that the distortionary effect of corruption will be more pronounced in case conflicting regions are asymmetric in terms of their benefits from water usages and/or costs of engaging in corrupt activities. This is because, in case of asymmetric regions, for any given rule of allocation, equilibrium contributions made by the two conflicting regions are likely to be different from each other, which will induce the corrupt planner to deviate from efficient allocation of water. Thus, in the case of asymmetric regions, the corrupt planner's choice of both (a) the rule of allocation and

(b) given the rule of allocation, levels of waters allocated to different regions are likely to be distorted. Nonetheless, it seems to be interesting to characterize the equilibrium in the case of asymmetric regions. It also seems to be interesting to extend the analysis by considering a dynamic game.

Appendix:

A1. Proof of Lemma 1

In stage 3, the problem of the central planer can be written as follows.

$$\max_{W_o \in (0, W_h)} E[Z]_F = \nu E\left[\left\{a(W - W_o) - \frac{b}{2}(W - W_o)^2\right\} + \lambda\left\{aW_o - \frac{b}{2}\,{W_o}^2\right\}\right]$$

$$+(1-\nu)E[0 + \lambda\left\{aW - \frac{b}{2}\,W^2\right\}]$$

Ignoring the boundary restrictions on W_o, the first order condition of the above problem yields $W_o = \frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)}$. Further, $\frac{\partial^2 E(Z)_F}{\partial {W_o}^2} = -b\,(1+\lambda) < 0$, which implies that the second order condition for maximization is satisfied. Note that (i) $\frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)} > 0 \Leftrightarrow b\,\overline{W} > a\,(1-\lambda)$, which is true since $\lambda \geq 0$ and $\overline{W} > \frac{a}{b}$; and (ii) $\frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)} < W_h \Leftrightarrow b(W_h - \overline{W}) + \lambda\,(b\,W_h - a) + a > 0$, which is always true, since $W_h > \overline{W}$ and $W_h > \frac{a}{b}$. It follows from (i) and (ii) that $\frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)} \in (0,\, W_h)$. Therefore, the solution of the above problem is given by $W_o = \frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)} = W_o(\lambda)$.

Now, since $W_o = W_o(\lambda)$, we have

$W_L = \begin{cases} W_o(\lambda), & if\ \ W_o(\lambda), < W \\ W, & if\ \ W_o(\lambda), \geq W \end{cases}$ and $W_U = \begin{cases} W - W_o(\lambda)\,, & if\ \ W_o(\lambda), < W \\ 0, & if\ \ W_o(\lambda), \geq W \end{cases}$ (from (3)), and $Prob(W_0(\lambda) < W) = \nu = \begin{cases} 1, & if\ W_o(\lambda) < W_l \\ \rho, & if\ \ W_l \leq W_0(\lambda) < W_h \end{cases}$. It follows that $E[B_L(W)\,|W_o = W_o(\lambda)] = \nu\, B_L(W_o(\lambda)) + (1-\nu)E[B_L(W)]$, and $E[B_U(W)\,|W_o = W_o(\lambda)] = \nu\, EB_U\big(W - W_o(\lambda)\big)]$.

[QED]

A2. Proof of Lemma 2

Given a fixed allocation rule, in the second stage the objective functions of the upstream region and the downstream region, respectively, can be written as follows.

$E[G_{U,F}] = \nu\left[\frac{2a\bar{W}\lambda^2}{(1+\lambda)^2} + \frac{b(1+2\lambda)\bar{W}^2}{2(1+\lambda)^2} + \frac{a^2(1-\lambda)}{b(1+\lambda)} - \frac{a^2(1-\lambda)^2}{2b(1+\lambda)^2} - \frac{bE(W^2)}{2}\right] - \psi\frac{C_U{}^2}{2}$ and

$E[G_{L,F}] = \nu\left[\frac{2a\bar{W}}{(1+\lambda)^2} - \frac{a^2(1-\lambda)(3+\lambda)}{2b(1+\lambda)^2} - \frac{b\bar{W}^2}{2(1+\lambda)^2}\right] + (1-\nu)\left[a\bar{W} - \frac{bE(W^2)}{2}\right] - \psi\frac{C_L{}^2}{2}$,

where $\lambda = 1 + C_L - C_U$.

Therefore, $\frac{\partial E[G_{U,F}]}{\partial C_U} = \nu\frac{\lambda(2a-b\bar{W})^2}{b(1+\lambda)^3} - \psi C_U$, $\frac{\partial}{\partial C_L}\left[\frac{\partial E[G_{U,F}]}{\partial C_U}\right] = \nu\frac{(1-2\lambda)(2a-b\overline{W})^2}{b(1+\lambda)^4}$, $\frac{\partial E[G_{L,F}]}{\partial C_L} = \frac{\nu\,(2a-b\bar{W})^2}{b(1+\lambda)^3} - \psi C_L$ and $\frac{\partial}{\partial C_U}\left[\frac{\partial E[G_{L,F}]}{\partial C_L}\right] = \nu\frac{3(2a-b\overline{W})^2}{b(1+\lambda)^4}$. It follows that

a) $\frac{\partial}{\partial C_L}\left[\frac{\partial E[G_{U,F}]}{\partial C_U}\right]\begin{cases} > 0, & if\ \lambda < \frac{1}{2} \Leftrightarrow \mathrm{C_U} > \mathrm{C_L} + \frac{1}{2} \\ < 0, & if\ \lambda > \frac{1}{2} \Leftrightarrow \mathrm{C_U} < \mathrm{C_L} + \frac{1}{2} \\ = 0, & if\ \lambda = \frac{1}{2} \Leftrightarrow \mathrm{C_U} = \mathrm{C_L} + \frac{1}{2} \end{cases}$ and

b) $\frac{\partial}{\partial C_U}\left[\frac{\partial E[G_{L,F}]}{\partial C_L}\right] > 0$ always holds true, since $\nu = \{1, \rho\}$ and $\rho \in (0,1)$.

[QED]

A3. Proof of Lemma 3

The problem of region i is $\max_{C_i \in [0,1]} E[G_{i,F}]$, $i = U, L$. First order conditions of these two maximization problems are as follows.

Upstream Region: $\frac{\partial E[G_{U,F}]}{\partial C_U} = \nu\frac{\lambda(2a-b\bar{W})^2}{b(1+\lambda)^3} - \psi C_U = 0$,

Downstream Region: $\frac{\partial E[G_{L,F}]}{\partial C_L} = \frac{\nu\,(2a-b\bar{W})^2}{b(1+\lambda)^3} - \psi C_L = 0$, where $\lambda = 1 + C_L - C_U$.

Solving the above first order conditions, we get $C_U = C_L = \nu\frac{(2a-b\bar{W})^2}{8b\psi}$.

Next, it is easy to check that (a) $\left.\frac{\partial^2 E[G_{U,F}]}{\partial C_U{}^2}\right|_{C_U=C_L} = \nu\frac{(2\mathrm{a}-\mathrm{b}\overline{\mathrm{W}})^2}{16\mathrm{b}} - \psi < 0 \Leftrightarrow \psi > \nu\frac{(2\mathrm{a}-\mathrm{b}\overline{\mathrm{W}})^2}{16\mathrm{b}}$, which is true by construction since $\nu \in (0,1]$, and (b) $\frac{\partial^2 E[G_{L,F}]}{\partial C_L{}^2} = \nu\frac{-3(2\mathrm{a}-\mathrm{b}\overline{\mathrm{W}})^2}{\mathrm{b}(1+\lambda)^4} - \psi < 0\ \forall\ \lambda \geq 0$. Therefore, second order conditions for maximization are satisfied.

Further, we have $\frac{\partial}{\partial C_U}\left[\frac{\partial E[G_{L,F}]}{\partial C_L}\right] = \nu \frac{3(2a-b\overline{W})^2}{b(1+\lambda)^4} > 0$ and $\frac{\partial}{\partial C_L}\left[\frac{\partial E[G_{U,F}]}{\partial C_U}\right]\Big|_{C_U=C_L} = \nu \frac{(1-2\lambda)(2a-b\overline{W})^2}{b(1+\lambda)^4}\Big|_{C_U=C_L} = -\nu \frac{(2a-b\overline{W})^2}{16b} < 0$. Thus, at $C_U = C_L$, $|H| = \begin{vmatrix} \frac{\partial^2 E[G_{U,F}]}{\partial {C_U}^2} & \frac{\partial}{\partial C_L}\left(\frac{\partial E[G_{U,F}]}{\partial C_U}\right) \\ \frac{\partial}{\partial C_U}\left(\frac{\partial E[G_{L,F}]}{\partial C_L}\right) & \frac{\partial^2 E[G_{L,F}]}{\partial {C_L}^2} \end{vmatrix} > 0$ holds true. It implies that the equilibrium is stable.

[QED]

A4. Proof of Lemma 4

We have $C_{fixed} = C_U = C_L = \nu \frac{(2a-b\overline{W})^2}{8b\psi}$. Thus, $\lambda = 1 + C_L - C_U = 1$. Now,

$W_o|_{\lambda=1} = \frac{b\,\overline{W} - a\,(1-\lambda)}{b\,(1+\lambda)}\Big|_{\lambda=1} = \frac{\overline{W}}{2}$. Clearly, $W_0^* = \frac{\overline{W}}{2}$.

It follows that, at the SPNE, expected water flow to the downstream region is given by $E[W_{L,F}]^* = \nu W_o|_{\lambda=1} + (1-\nu)\overline{W} = \overline{W}(1-\frac{\nu}{2})$ and expected water share of the upstream region is given by $E[W_{U,F}]^* = \nu[\overline{W} - W_o|_{\lambda=1}] + (1-\nu)0 = \nu\frac{\overline{W}}{2}$. The central planner's SPNE payoff is given by $O_F^* = 2C_{fixed} = \nu\frac{(2a-b\overline{W})^2}{4b\psi}$.

[QED]

A5. Proof of Lemma 5

Under the proportional allocation rule, the planner's problem in stage 3 is given by $\max_{\beta\in(0,1)} E[Z]_P$, where

$$E[Z]_P = E[B_U\{(1-\beta)W\}] + \lambda\, E[B_L(\beta W)]$$

$$= E[a(1-\beta)W - \frac{b}{2}\{(1-\beta)W\}^2] + \lambda\, E\left[a\beta W - \frac{b}{2}\,(\beta W)^2\right].$$

The first order condition of the central planner's stage 3 problem is

$$\frac{\partial E[Z]_P}{\partial \beta} = -a\overline{W} + b(1-\beta)E(W^2) + \lambda\,[a\overline{W} - b\beta E(W^2)\,] = 0 \Rightarrow \beta = \frac{bE(W^2) - a(1-\lambda)\overline{W}}{b(1+\lambda)E(W^2)} = \beta(\lambda).$$

Now, $\frac{\partial^2 E(Z)_P}{\partial \beta^2} = -b(1+\lambda)E(W^2) < 0$, which implies that the second order condition for maximization is satisfied. It follows that $E[Z]_P$ is strictly concave in β and has a unique maximum at $\beta = \beta(\lambda)$.

Now, $\beta(\lambda) \geq 1 \Leftrightarrow \frac{bE(W^2)-a(1-\lambda)\bar{W}}{b(1+\lambda)E(W^2)} \geq 1 \Leftrightarrow \lambda[a\bar{W} - bE(W^2)] \geq a\bar{W}$, which is possible to hold only if $a\bar{W} - bE(W^2) > 0$, since $a > 0$, $\bar{W} > 0$ and $\lambda \geq 0$. If $[a\bar{W} - bE(W^2)] > 0$, $\beta(\lambda) \geq 1 \Leftrightarrow \lambda \geq \frac{a\bar{W}}{[a\bar{W} - bE(W^2)]} = \bar{\lambda}$.

Next, $\beta(\lambda) \leq 0 \Leftrightarrow \frac{bE(W^2)-a(1-\lambda)\bar{W}}{b(1+\lambda)E(W^2)} \leq 0 \Leftrightarrow \lambda[a\bar{W}] \leq a\bar{W} - bE(W^2)$, which is possible to hold only if $a\bar{W} - bE(W^2) > 0$, since $a > 0$, $\bar{W} > 0$ and $\lambda \geq 0$. If $[a\bar{W} - bE(W^2)] > 0$, $\beta(\lambda) \leq 0 \Leftrightarrow \lambda \leq \frac{[a\bar{W} - bE(W^2)]}{a\bar{W}} = \underline{\lambda}$.

So, we have the following.

(a) If $[a\bar{W} - bE(W^2)] \leq 0$, $0 < \beta(\lambda) < 1$ and, thus, $\beta^* = \beta(\lambda)$.

(b) If $[a\bar{W} - bE(W^2)] > 0$ and $\underline{\lambda} < \lambda < \bar{\lambda}$, $0 < \beta(\lambda) < 1$ and, thus, $\beta^* = \beta(\lambda)$.

(c) If $[a\bar{W} - bE(W^2)] > 0$ and $\underline{\lambda} \geq \lambda$, $\beta(\lambda) \leq 0$, which implies that $\beta^* = \epsilon$, where ϵ is a very small positive number.

(d) If $[a\bar{W} - bE(W^2)] > 0$ and $\lambda \geq \bar{\lambda}$, $\beta(\lambda) \geq 1$, which implies that $\beta^* = 1 - \epsilon$.

[QED]

A6. Proof of Lemma 6

Objective functions of upstream and downstream regions in stage 2 of the game under proportional allocation rule can be written as follows, where $\lambda = 1 + C_L - C_U$.

Upstream Region: $E[G_{U,P}] = \frac{2a\bar{W}\lambda^2}{(1+\lambda)^2} + \frac{a^2(1-\lambda)(3\lambda+1)\bar{W}^2}{2b(1+\lambda)^2 E(W^2)} - \frac{bE(W^2)\lambda^2}{2(1+\lambda)^2} - \psi\frac{{C_U}^2}{2}$

Downstream Region: $E[G_{L,P}] = \frac{2a\bar{W}}{(1+\lambda)^2} - \frac{a^2(1-\lambda)(3+\lambda)\bar{W}^2}{2b(1+\lambda)^2 E(W^2)} - \frac{bE(W^2)}{2(1+\lambda)^2} - \psi\frac{{C_L}^2}{2}$

It is easy to check that $\frac{\partial E[G_{U,P}]}{\partial C_U} = \frac{\lambda(2a\bar{W}-bE(W^2))^2}{b(1+\lambda)^3\, E(W^2)} - \psi C_U$, $\frac{\partial}{\partial C_L}\left[\frac{\partial E[G_{U,P}]}{\partial C_U}\right] = \frac{(1-2\lambda)(2a\bar{W}-bE(W^2))^2}{b(1+\lambda)^4 E(W^2)}$, $\frac{\partial E[G_{L,P}]}{\partial C_L} = \frac{(2a\bar{W}-bE(W^2)^2}{b(1+\lambda)^3} - \psi C_L$ and $\frac{\partial}{\partial C_U}\left[\frac{\partial E[G_{L,P}]}{\partial C_L}\right] = \frac{3(2a\bar{W}-bE(W^2))^2}{b(1+\lambda)^4}$. Clearly,

(a) $\frac{\partial}{\partial C_U}\left[\frac{\partial E[G_{L,P}]}{\partial C_L}\right] > 0$ always holds true, but

(b) $\frac{\partial}{\partial C_L}\left[\frac{\partial E[G_{U,P}]}{\partial C_U}\right] \begin{cases} > 0, & if\ \lambda < \frac{1}{2} \Leftrightarrow \mathrm{C_U} > \mathrm{C_L} + \frac{1}{2} \\ < 0, & if\ \lambda > \frac{1}{2} \Leftrightarrow \mathrm{C_U} < \mathrm{C_L} + \frac{1}{2} \\ = 0, & if\ \lambda = \frac{1}{2} \Leftrightarrow \mathrm{C_U} = \mathrm{C_L} + \frac{1}{2} \end{cases}.$

[QED]

A7. Proof of Lemma 7

The problem of region i in stage 2 of the game can be written as follows.

$$\max_{C_i \in [0,1]} E[G_{i,P}], \qquad i = U, L;$$

where $E\left[G_{U,P}\right] = \frac{2a\bar{W}\lambda^2}{(1+\lambda)^2} + \frac{a^2(1-\lambda)(3\lambda+1)\bar{W}^2}{2b(1+\lambda)^2 E(W^2)} - \frac{bE(W^2)\lambda^2}{2(1+\lambda)^2} - \psi\frac{{C_U}^2}{2}$,

$E[G_{L,P}] = \frac{2a\bar{W}}{(1+\lambda)^2} - \frac{a^2(1-\lambda)(3+\lambda)\bar{W}^2}{2b(1+\lambda)^2 E(W^2)} - \frac{bE(\bar{W}^2)}{2(1+\lambda)^2} - \psi\frac{{C_L}^2}{2}$ and $\lambda = 1 + C_L - C_U$.

Thus, first order conditions of upstream and downstream regions maximization problems are, respectively, given by

$\frac{\partial E[G_{U,P}]}{\partial C_U} = \frac{\lambda(2a\bar{W}-bE(W^2))^2}{b(1+\lambda)^3\, E(W^2)} - \psi C_U = 0$ and $\frac{\partial E[G_{L,P}]}{\partial C_L} = \frac{(2a\bar{W}-bE(W^2)^2}{b(1+\lambda)^3} - \psi C_L$; where $\lambda = 1 + C_L - C_U$. Solving these two first order conditions we get $C_L = C_U = \frac{[2a\bar{W}-bE(W^2)]^2}{8b\psi E(W^2)} = C_{pro}$, say.

Next, it is easy to check that

(a) $\frac{\partial^2 E[G_{L,P}]}{\partial {C_L}^2} = \frac{-3(2a\bar{W}-b\bar{W})^2}{b(1+\lambda)^4} - \psi < 0$

(b) $\frac{\partial^2 E[G_{U,P}]}{\partial {C_U}^2}\Big|_{C_U=C_L} = \frac{(2a\bar{W}-bE(W^2))^2}{16bE(W^2)} - \psi < 0$, since $\psi \geq \frac{(2a\bar{W}-bE(W^2))^2}{8bE(W^2)}$ by Assumption.

(c) Whenever $C_U = C_L$ is satisfied, $\left|H^{/}\right| = \begin{vmatrix} \frac{\partial^2 E[G_{U,P}]}{\partial {C_U}^2} & \frac{\partial}{\partial C_L}\left(\frac{\partial E[G_{U,P}]}{\partial C_U}\right) \\ \frac{\partial}{\partial C_U}\left(\frac{\partial E[G_{L,P}]}{\partial C_L}\right) & \frac{\partial^2 E[G_{L,P}]}{\partial {C_L}^2} \end{vmatrix} > 0$; since

$\left[\frac{\partial}{\partial C_L}\left(\frac{\partial E[G_{U,P}]}{\partial C_U}\right)\right]_{C_U=C_L} = \left.\frac{(1-2\lambda)(2a\overline{\mathrm{W}}-bE(W^2))^2}{bE(W^2)(1+\lambda)^4}\right|_{C_U=C_L} = -\frac{\left(2\mathrm{a}\overline{\mathrm{W}}-\mathrm{b}E(W^2)\right)^2}{16\mathrm{b}E(W^2)} < 0$,

$\left[\frac{\partial}{\partial C_U}\left(\frac{\partial E[G_{L,P}]}{\partial C_L}\right)\right] = \frac{3(2a\overline{\mathrm{W}}-bE(W^2))^2}{b(1+\lambda)^4} > 0$, $\frac{\partial^2 E[G_{L,P}]}{\partial {C_L}^2} < 0$ and $\left.\frac{\partial^2 E[G_{U,P}]}{\partial {C_U}^2}\right|_{C_U=C_L} < 0$.

It follows from (a), (b) and (c) that second order conditions for maximization are satisfied and the equilibrium is stable. Thus, the equilibrium contributions made by regions are given by $C_{L,P} = C_{U,P} = \frac{[2a\bar{W}-bE(W^2)]^2}{8b\psi E(W^2)} = C_{pro}$.

[QED]

A8. Proof of Lemma 8

From Lemma 7, $C_L = C_U = \frac{[2a\bar{W}-bE(W^2)]^2}{8b\psi E(W^2)} = C_{pro}$, which implies that $\lambda = \lambda^* = 1$. Substituting $\lambda = 1$ in the expression for β(λ) in Lemma 5, we get $\beta(\lambda)|_{\lambda=1} = \frac{1}{2}$. Therefore, $\beta = \frac{1}{2} = \beta^*$ is the SPNE proportion of water for the downstream region.

It follows that $E[W_{L,P}]^* = \beta^* E[W] = \frac{\bar{W}}{2}$ and $E[W_{U,P}]^* = (1-\beta^*)E[W] = \frac{\bar{W}}{2}$.

Now, since $O_P = C_L + C_U$ and in the SPNE $C_L = C_U = \frac{[2a\bar{W}-bE(W^2)]^2}{8b\psi E(W^2)} = C_{pro}$, we have $O_P^* = 2C_{pro} = \frac{[2a\bar{W}-bE(W^2)]^2}{4b\psi E(W^2)}$.

[QED]

A9. Proof of Proposition 1

We have the following.

$O_F^* = 2\ C_{fixed} = \nu\,\frac{(2a-b\bar{W})^2}{4b\psi}$ (from Lemma 4) and

$O_P^* = 2C_{pro} = \frac{[2a\bar{W} - bE(W^2)]^2}{4b\psi E(W^2)}$ (from Lemma 8).

Note that the central planner chooses the fixed allocation rule if and only if $O_F^* > O_P^*$. Otherwise, if $O_F^* < O_P^*$, the proportional allocation rule is the central planner's optimal choice.

Now,

$$O_F^* - O_P^* > 0 \Leftrightarrow \nu > \frac{[2a\bar{W} - bE(W^2)]^2}{(2a - b\bar{W})^2 \, E(W^2)} = \hat{\nu} \qquad (A1)$$

Clearly, $\hat{\nu} > 0$. The following is immediate from (A1).

i) If $\hat{\nu} > 1$, condition (A1) can never be satisfied, since $\nu \in \{1, \rho\}$ and $\rho \in (0, 1)$. In other words, if $\hat{\nu} > 1$, we must have $O_F^* < O_P^*$.

ii) If $\hat{\nu} < 1$, then the following is true.

   a) $O_F^* > O_P^*$ when $\nu > \hat{\nu}$.

   b) $O_F^* < O_P^*$ when $\nu < \hat{\nu}$.

Now, it is easy to check that

$\hat{\nu} < 1 \Leftrightarrow \frac{[2a\bar{W} - bE(W^2)]^2}{E(W^2)} < (2a - b\bar{W})^2 \Leftrightarrow \frac{4a^2}{b^2} > E(W^2) = \bar{W}^2 + \sigma^2$ and

$\hat{\nu} > 1 \Leftrightarrow \frac{4a^2}{b^2} < \bar{W}^2 + \sigma^2$.

Water Abundance ($\bar{W} \geq \frac{2a}{b}$)

It is evident that, if $\bar{W} \geq \frac{2a}{b}$, $\frac{4a^2}{b^2} < \bar{W}^2 + \sigma^2$ always holds true. Therefore, $\hat{\nu} > 1$. It implies that, if $\bar{W} \geq \frac{2a}{b}$, we must have $O_F^* < O_P^*$. Thus, in the case of water abundance, the corrupt central planner strictly prefers the proportional allocation rule over the fixed allocation rule.

Severe and Moderate Water Scarcity ($\bar{W} < \frac{2a}{b}$)

When $\bar{W} < \frac{2a}{b}$, $\hat{\nu} \gtreqless 1$ depending on the magnitude of $\bar{W}$ for given values of a, b and σ^2. Now, $\hat{\nu} > 1 \Leftrightarrow \frac{4a^2}{b^2} < \bar{W}^2 + \sigma^2 \Leftrightarrow \bar{W} > \sqrt{\frac{4a^2}{b^2} - \sigma^2} = \widetilde{W}$, say. Therefore, if $\widetilde{W} < \bar{W} < \frac{2a}{b}$, $O_F^* < O_P^*$ holds true.

It follows that (a) $\hat{\nu} < 1 \Leftrightarrow \bar{W} < \widetilde{W}$ and (b) $\hat{\nu} = 1 \Leftrightarrow \bar{W} = \widetilde{W}$.

Note that $[\nu = 1] \Leftrightarrow [Prob(W > W_0) = 1] \Leftrightarrow [W_0 < W_l]$. That is, if (a) the central planner sets the fixed amount W_0 such that $W_0 < W_l$ and (b) $\bar{W} = \widetilde{W}$, then the corrupt planner is indifferent between the two rules of allocation, i.e. $O_F^* = O_P^*$. Next, under fixed allocation rule the central planner's equilibrium choice of W_0 is given by $W_0 = \frac{\bar{W}}{2}$. It follows that $O_F^* = O_P^*$ holds only if $\bar{W} = \widetilde{W} < 2W_l$; otherwise, at $\bar{W} = \widetilde{W}$, we have $O_F^* < O_P^*$.

Since $W_0 = \frac{\bar{W}}{2}$ in the equilibrium under fixed allocation rule, $\nu = Prob(W > W_0) = Prob(\bar{W} < 2W) = 1$ in the equilibrium, iff $\bar{W} < 2\,W_l$. Note that we have $2\,W_l < \frac{2a}{b}$, by construction.

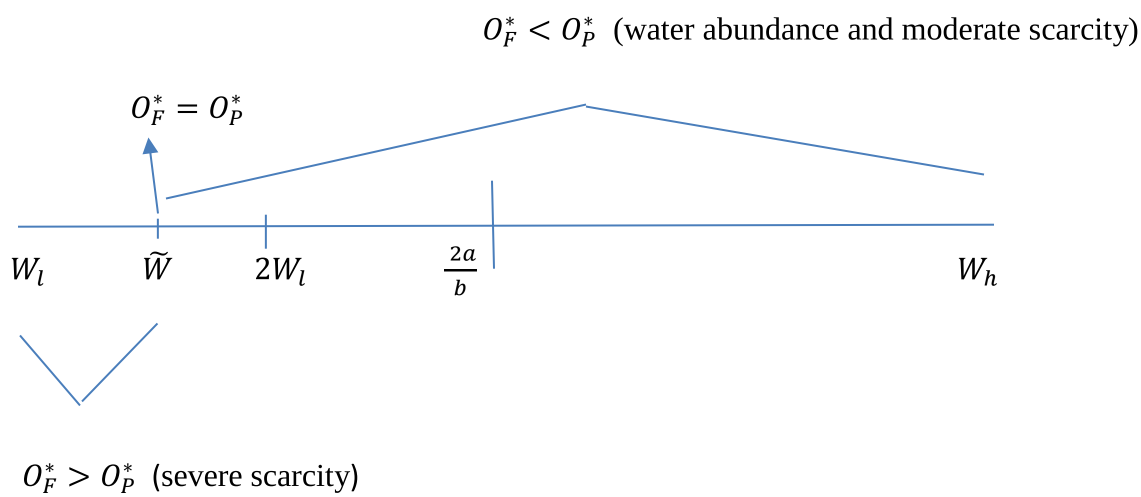
Case I: $\mathbf{2W_l > \widetilde{W}}$

(a) If $2W_l > \widetilde{W}$, then for all $\bar{W} \in (\widetilde{W}, 2\,W_l)$ (i) $\nu = 1$, since $W_0 = \frac{\bar{W}}{2} < W_l$, and (ii) $\hat{\nu} > 1$, since $\bar{W} > \widetilde{W}$. Thus, $\nu < \hat{\nu}$ holds true.
(b) If $\bar{W} = 2\,W_l$, $W_0 = \frac{\bar{W}}{2} = W_l$ and $\nu = Prob(W > W_0) = Prob(W = W_h) = \rho < 1$. But we must have $\hat{\nu} > 1$ since $\bar{W} > \widetilde{W}$. Thus, $\nu < \hat{\nu}$ holds true.

From (a) and (b), we have $\forall\, \bar{W} \in (\widetilde{W}, \frac{2a}{b})$, $O_F^* < O_P^*$, since $\nu < \hat{\nu}$ holds true.

Now, if $\bar{W} = \widetilde{W}$, then $O_F^* = O_P^*$ since $\bar{W} = \widetilde{W} < 2W_l$.

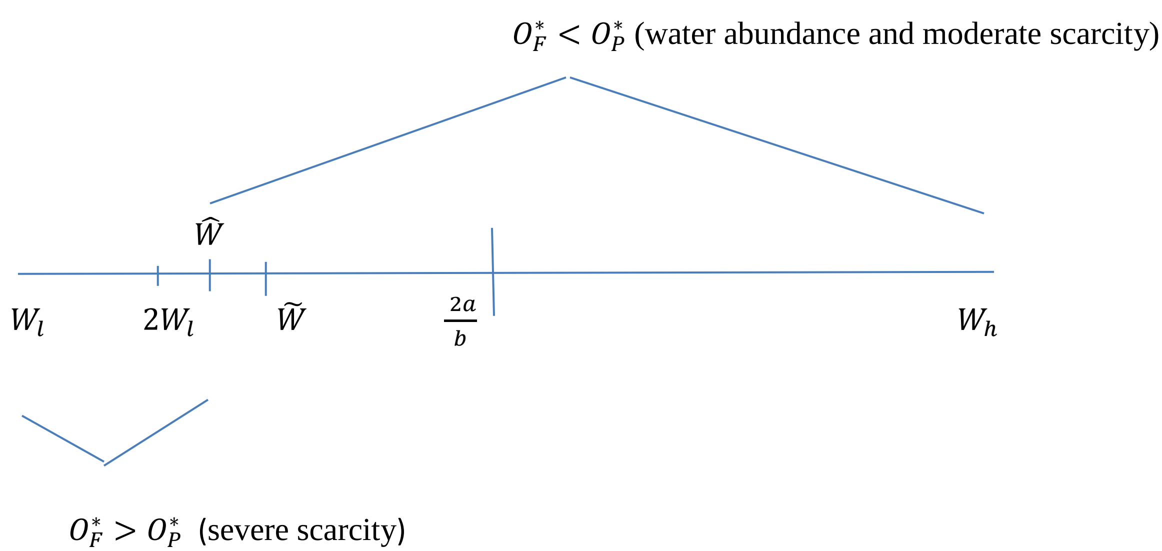
And, finally, if $\bar{W} < \widetilde{W}$, we still have $\nu = 1$, since $W_0 = \frac{\bar{W}}{2} < W_l$, but $\hat{\nu} < 1$, since $\bar{W} < \widetilde{W}$. So, $\forall\, \bar{W} \in (W_l, \widetilde{W})$, $O_F^* > O_P^*$.



Overall, we get the following.

1. $O_F^* > O_P^*, \forall\, \bar{W} \in (W_l,\ \tilde{W})$,
2. $O_F^* < O_P^*$, $\forall\, \bar{W} \in (\tilde{W}, \frac{2a}{b})$, and
3. $O_F^* = O_P^*$, if $\bar{W} = \hat{W}$, where $\tilde{W} < 2W_l < \frac{2a}{b}$

$Case\ II$: $\boldsymbol{2W_l < \tilde{W}}$



Now, we have to prove the following claim:

Claim: *There exists a unique* $\widehat{W} \in (2W_l,\ \widetilde{W})$*, such at (a)* $\forall\, \overline{W} \in (2W_l,\ \widehat{W}),\ \nu > \hat{\nu}$*, and (b)* $\nu < \hat{\nu}\ \forall\, \overline{W} \in (\widehat{W}, \widetilde{W})$.

Proof:

We have the following.

i) $\nu = \rho \in (0,1), \quad \forall\, \overline{W} \in (2W_l,\ \widetilde{W}]$. This is because, $\nu = Prob(W > W_0) = Prob(W = W_h) = \rho$, $\widetilde{W} < \frac{2a}{b} < 2W_h$, and in the equilibrium under fixed allocation rule $W_0 = \frac{\overline{W}}{2}$.

ii) $\hat{\nu} = 1$, if $\overline{W} = \widetilde{W}$.

It can be checked that

$$\hat{\nu} < (=)\rho \ \Leftrightarrow \overline{W}^2 < (=) \left[\frac{\rho\sigma^2}{1 - \rho + \frac{\sigma^2 b^2}{(2a - b\overline{W})^2}} \right] = [\overline{W}(\rho)]^2$$

It is evident that $\overline{W}(\rho) < \overline{W}(1)$. It follows that $\hat{\nu}$ is increasing in $\overline{W}$ for any given σ^2.

Clearly, there exists a unique $\widehat{W} \in (2W_l,\ \widetilde{W})$, such that (a) $\hat{\nu} = \rho = \nu$ when $\overline{W} = \widehat{W}$, (b) $\nu > \hat{\nu}\ \forall\, \overline{W} \in (2W_l,\ \widehat{W})$ and (c) $\nu < \hat{\nu}\ \forall\, \overline{W} \in (\widehat{W}, \widetilde{W})$.

Overall, we get the following.

1. $O_F^* > O_P^*$, $\forall\, \overline{W} \in (W_l,\ \widehat{W})$,
2. $O_F^* < O_P^*$, $\forall\, \overline{W} \in (\widehat{W}, \frac{2a}{b})$, and
3. $O_F^* = O_P^*$, if $\overline{W} = \widehat{W}$, where $2W_l < \widehat{W} < \widetilde{W} < \frac{2a}{b}$

$Case\ III$: $\mathbf{2W_l = \widetilde{W}}$

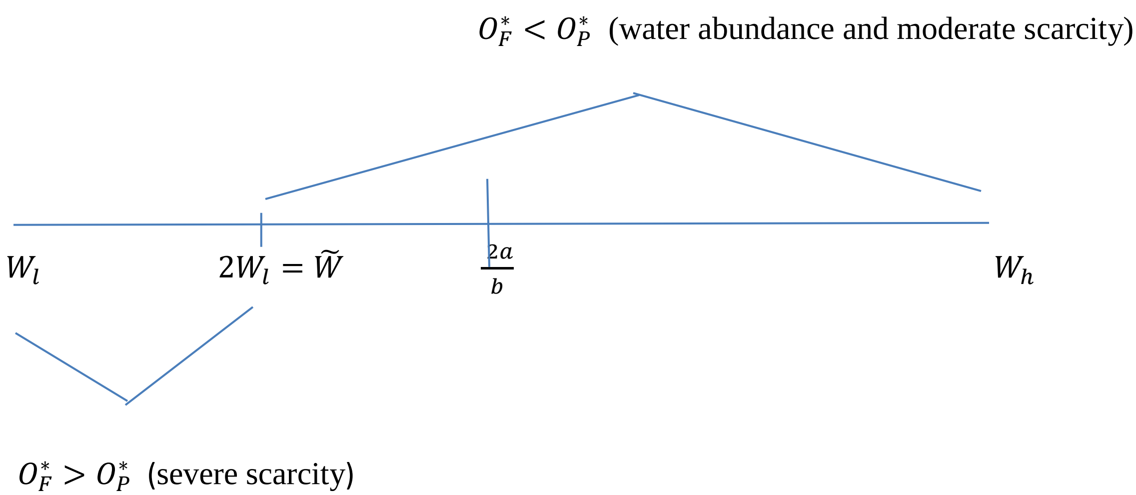
If $\overline{W} = \widetilde{W} = 2\,W_l$, $W_0 = \frac{\overline{W}}{2} = W_l$ and $\nu = Prob(W > W_0) = Prob(W = W_h) = \rho < 1$.

But we must have $\hat{\nu} = 1$ since $\overline{W} = \widetilde{W}$. Thus, $\nu < \hat{\nu}$ holds true.

So, we have for $\overline{W} = \widetilde{W} = 2\,W_l$, $O_F^* < O_P^*$, since $\nu < \hat{\nu}$.

For $\overline{W} > \widetilde{W} = 2\,W_l$, we have $O_F^* < O_P^*$ as $\nu < \hat{\nu}$, since $\nu = Prob(W > W_0) = Prob(W = W_h) = \rho < 1$ and $\hat{\nu} > 1$.

For $\overline{W} < \widetilde{W} = 2\,W_l$, we have $O_F^* > O_P^*$ as $v > \hat{v}$, since $\nu = 1$ and $\hat{v} < 1$.



Therefore, we can state the following.

It is optimal for the corrupt central planner to choose the fixed allocation rule, if either of the following is true.

Case I: $\overline{W} \in (W_l, \widetilde{W}\ )$, where $W_l < \widetilde{W} < 2W_l$

Case II: $\overline{W} \in (W_l,\ \widehat{W})$, where $2W_l < \widehat{W} < \sqrt{\frac{4a^2}{b^2} - \sigma^2}\ = \widetilde{W}$

Case III: $\overline{W} \in [W_l\ , 2W_l = \widetilde{W})$, where $\overline{W} = \widetilde{W} = 2\,W_l$.

It implies that there exists a $W = \underline{W}$, where $\underline{W} \leq \widetilde{W} = \sqrt{\frac{4a^2}{b^2} - \sigma^2}$, such that if $\overline{W} \in (W_l, \underline{W}\ )$, it is optimal for the corrupt central planner to choose the fixed allocation rule. Otherwise, if $\overline{W} \geq \frac{2a}{b}$ or $\overline{W} \in \left[\underline{W}, \frac{2a}{b}\right)$, it is optimal for the corrupt social planner to choose the proportional allocation rule.

[QED]

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