WP-2021-003

Peace over War: Conflict, Contest and Cooperation in Water Sharing

Rupayan Pal and Dipti Ranjan Pati

Indira Gandhi Institute of Development Research, Mumbai
January 2021

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Email(corresponding author): rupayan@igidr.ac.in

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Keywords: Contest success function; Cooperation; Endogenous claims; Water conflicts; Property rights

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Corresponding Author and Address:

Rupayan Pal, Indira Gandhi Institute of Development Research (IGIDR), Film City Road, Goregaon (E), Mumbai – 400065, India. Phone: +91-22-28416545.

Emails: (Pal) rupayan@igidr.ac.in, rupayanpal@gmail.com; searchme.dipti@gmail.com.

1. Introduction

Shared water always and everywhere carries the seed of conflict. But it is also the very nature of this resource that makes it so amenable to sharing. As Wolf, Yoffe and Giordano (2003) write, "…international water may be as a resource whose characteristics tend to induce cooperation, and incite violence only in the exception." Investigating historic water conflicts, Wolf (1999) finds that war over water doesn't seem strategically rational, hydrographically effective or economically viable. The same sense resonates in Chokkakula (2014), who notes that in relation to a few interstate water disputes in India, there are more than 120 interstate water agreements.

Conflict resolution sometimes follows formal processes of negotiation and any cooperative outcome, then, is explicitly attributed to this process. Wolf(1999) notes that such cooperative regimes established through treaty turn out to be quite resilient over time, even between otherwise hostile riparians who continue to spar over other issues. Sometimes, however, solutions to potentially intractable and messy water conflicts seem to appear seamlessly, in which case, one must look into the implicit driving mechanisms. We posit that this seemingly cooperative outcome of many water conflicts may simply be the result of a non-cooperative game with an efficient solution that plays out silently in the background. This paper tries to go into the heart of such a potential water conflict where the efficient solution obtains without negotiation or any explicit intervention in the form of either arbitration or political diktat and tries to locate water conflict within the larger scope of contest over limited resources.

We track closely a setup straddling two areas of conflict literature, namely water as a source of conflict and the use of contest technology as a means of resolution, and explore the possibility of an efficient outcome. We note here that such a setup is used by Ansink and Weikard (2009) to show the inefficiency of contest over water. There are two riparian regions with fixed endowments and claims on as-of-yet unallocated water. We deal with the allocation of that water which is claimed by both and call it contested water. Their claims are not exogenous but contingent on the system capacity, or complementary infrastructure as we call it, that they have built prior to allocation. We make no distinction between system capacity and beneficial use in our model as we assume operation at full capacity. Also, following Johnson, Gisser and Werner (1981), we assume that property rights over water are to be defined in terms of consumptive use and not diversion.

The fact that a region has built system capacity or created avenues for beneficial use prior to water sharing makes its claims all the more credible. This legitimization of claims on the basis of built infrastructure, through a sharing rule that is common knowledge, introduces a semblance of appropriative property rights into our paradigm. Although it mimics the appropriative rights regime, it is still not a property right proper as we also allow our regions to invest part of their endowment towards a fighting input which they can use to secure the contested water. The share of water obtained through the process is modelled in terms of a Contest Success Function (CSF). We first consider a linear CSF, with a particular functional characterization, and then we extend the analysis to allow for a general CSF. This also allows us to vary the degree of mediation involved in the contest by first requiring anonymity and then, loosening the setup. Under such a paradigm, using the particular functional form, we obtain a non-cooperative result which is the same as the cooperative game outcome where no resource is spent on fighting, and thus, is Pareto efficient. This outcome persists irrespective of whether regions are symmetric or not in their endowments and productivities. This efficient outcome is obtained for both simultaneous and sequential move games. It is important to note that such an outcome obtains without any bargaining. We also obtain allocative efficiency under certain parametric conditions. Moreover, by treating water conflicts within the ambit of contest literature, our efficiency result is found to be more universal than many seminal works such as Skaperdas (1992) that explore the possibility of cooperation materializing through the use of conflict technology. With the more generalised forms of productive and contest technologies, we obtain conflict and cooperation to varying degrees.

Burness and Quirk (1979) find that in the absence of a competitive market in water rights, the doctrine of prior appropriation leads to inefficient use of water. This is mainly due to the recognition of 'seniority rights' which leads to unequal risk sharing even between firms having identical production functions. They contend that equal sharing of water, which would remove this asymmetry of risk sharing, would lead to problems that are faced by common property resources and hence, would not comport with a competitive market. We find that for regions with identical production functions, the non-cooperative water sharing outcome is allocatively efficient, even when the claims are endogenized in some form akin to the appropriative doctrine. This is evidently due to the fact that in our model, initially, we recognize no seniority rights and water is shared between similarly placed but not necessarily identical agents. Later, when we introduce seniority rights in the form of an explicit first mover advantage to the upstream region[1], we still obtain efficiency in a range of cases where the downstream region is well endowed and able to match the former's claims on water. But this framework yields outright or partial conflict,[2] if the comparative advantage of the upstream region is too high, or if the endowment of the downstream region is not sufficiently high. This is borne out repeatedly in riparian conflicts around the world where the upstream region often pre-emptively undertakes hydraulic projects on the upper reaches of a river, evidently to secure a significant comparative advantage. In case of transboundary flows where the upper riparian happens to enjoy relatively greater economic, military and geopolitical clout, such acts of pre-emptive infrastructure-building on its part bestow it with a significant comparative advantage, leaving the lower riparian with a limited recourse of politico-legal contestation. Similar behaviour is observed within federations, where upper riparians often initiate the pre-emptive diversion of water, to be followed by the lower riparian either matching such diversion or resorting to fighting through legal or political means, depending on the relative merits of its choices.

In India, for example, where most of the surface water derives from interstate rivers, conflict between states over water shares invariably erupts with the upper riparian pre-emptively diverting water. Although India recognizes some form of riparian rights, it is more of a regulated riparianism[3] that also blends elements of appropriation. So, states make claims to water on the basis of prior use so that investment in water infrastructure plays a crucial role. As Richards and Singh (2002) notice, states tend to deliberately delay the bargaining or adjudication process in order to build up capacity systems in the intervening period and thus, ratchet up their claims. In the notorious Cauvery water dispute, Karnataka, the upper riparian, had pre-emptively undertaken many hydraulic projects. In response, Tamil Nadu, the lower riparian, increased its irrigated acreage manifold and staked claim to a much greater share of water. It is crucial to note here that beneficial use, much like system capacity, can be trumped up to increase one's claims to water. But it has to be noted that in the Cauvery case, despite

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[1] We note here that there is nothing sacrosanct about the consideration that the upstream riparian enjoys a comparative advantage. Salman (2010) notes that downstream riparians could harm upstream riparians by foreclosing their future uses by ramping up projects and claiming rights based on prior use. Recognising this possibility, the Helsinki Rules on international water law require both upstream and downstream riparians to inform their counterparts of proposed installations on the shared river basin.

[2] Partial conflict denoting complete investment in productive infrastructure by one region and at least partial investment in conflict technology by the other

[3] Joseph W. Dellapenna, "Owning Water in the Eastern United States," 6 *E. Min. L. Inst*.

the veneer of implacable hostility due to the increasing politicization of the issue, there has been substantial cooperation between the two states, partly due to the fact that both are economically and politically powerful and there is no substantial first mover advantage. A similar dispute between the states of Chattisgarh and Odisha over the river Mahanadi has seen the former, which is the upper riparian, unilaterally building infrastructure which might serve to strengthen its bargaining position when the adjudicating tribunal decides the allocation. We note here that adjudication by a non-partisan tribunal roughly corresponds to a mediated contest where the rules and payoffs are common knowledge.

Trade in water is accepted as an efficient way of getting around the intractable problem of water conflict. But as Sampath (1992) and Richards and Singh (2001) observe, trade in water is hindered by the absence of pure private property rights over water. Fisher (1995), in an application to the conflict over water in the Middle East, attempts to bypass the issue of property rights altogether by invoking the Coase theorem which gives conditions under which efficiency doesn't depend upon the allocation of property rights. He suggests that since ownership of water is the same as a right to the monetary value of its use, it might be a good idea to look beyond the initial allocation of property rights and instead, allow the parties to trade in 'non-water' benefits. Richards and Singh (2001) observe that the Coase theorem holds only under certain conditions, namely, the absence of wealth effects and transaction costs, a condition not often met in the case of transnational water, which may prevent the separation of the issue of allocative efficiency of water from that of distribution of property rights. Also, since water markets must necessarily involve significant complementary investments in pipes and canals, the need for contract enforceability becomes all the more important as in its absence, such investments will not be undertaken. This paper allows for trade after having resolved the problem of initial allocation of property rights. We proceed from a situation where property rights over water are not defined, but the rules of the contest are known. This allows for rights to be defined through contest. The problem of enforceability is also taken care of in the mediated contest. Once such rights over water are defined, the regions are free to engage in trading and improve upon the game solution.

So, our paper deals with two main questions- one pertaining to the efficiency of non-cooperative games over water and another, to the effectiveness of contest technologies and their place in the larger contest literature.

Our paper follows in the lines of contest theory by assuming that players are locked in a state where property rights are absent or only half-baked and that players have an alternative recourse through a 'conflict technology' modelled using *Contest Success Functions (CSFs).* Hirshleifer (1998) explores the role of conflict technology in effecting cooperation when agents choose between production and appropriation as alternative means of generating income. Skaperdas (1992) finds similar results, with full cooperation, partial cooperation and conflict materializing under different levels of effectiveness of the conflict technology in operation.

While Skaperdas (1992) uses a generalized CSF satisfying certain axioms, Baik (1998) generalizes a difference-form CSF which satisfies similar axioms to arrive at similar results. It is worth noting here that our paper finds complete cooperation as a more universal result than existing literature. We do not consider joint production like Skaperdas (1992), but deal with separate benefit functions. Moreover, while the good under contest in existing contest literature happens to be the one jointly produced, in our paper, we consider separate benefit functions which use the contested good as an input. Ansink and Weikard (2009) also use separate production functions. We believe such a model to be eminently suitable for the water sharing case where regions derive different economic benefits and have different productivities.

The contest literature builds predominantly around the use of Contest Success Functions that give the probabilities of win or a sharing rule, given the efforts or inputs of contestants. Hirshleifer (1989) compares logistic and ratio CSFs and finds the former to possess certain characteristics that make it more suitable. Skaperdas (1996) characterizes a general CSF and finds that only the ratio and the logistic CSFs satisfy the related axioms. Skaperdas (1996) dismisses the Probit CSF used by Dixit (1987) as it doesn't satisfy the *'independence from irrelevant alternatives'* property. We use a 2-player linear CSF and then generalize it to a multi-player contest. We find that the linear CSF satisfies a particular characterization of the consistency axiom in Skaperdas (1996). It is also shown to satisfy the 'independence of irrelevant alternatives' property.

The rest of the paper is arranged in the following way. Section 2 delineates the model with the specific functional form and analyses the cooperative outcome. Section 3 generalises the productive and contest technologies and notes the spectrum of cooperation and conflict.

Section 4 explores cooperation in a situation with explicit comparative advantage and also shows that the linear CSF used by us cannot be dismissed as in Skaperdas (1996) as it satisfies 'consistency' and 'independence of irrelevant alternatives'. Section 5 concludes. Proofs are presented in the appendix.

2. A Simple Model

There are two regions 1 and 2 with overlapping claims to water. Total water is normalized to 1 (one). Since claims are overlapping, those add up to more than one and, so, water is contested. Region i $(= 1, 2)$ has endowment $e_i \in (0,1]$ of resources, which can be used for capacity-building complementary investment $x_i \in [0, e_i]$ and /or as fighting input $g_i \in [0, e_i]$. The endowment constraint of region i can be written as follows.

$$e_i = x_i + g_i\ , i = 1, 2 \tag{1}$$

The payoff of region i $(= 1,2)$ is considered given by

$$\pi_i = (x_i w_i)^{\alpha_i}\ ; \quad 0 < \alpha_i \leq 1 \ \ and \ \ i = 1,2; \tag{2}$$

where $w_i \in [0,1]$ and α_i denote, respectively, water share and productivity parameter of region i. Higher value of α_i indicates higher productivity of w_i and x_i. Note that (a) $\pi_i(0, 0) = \pi_i(x_i, 0) = \pi_i(0, w_i) = 0$ and

(b) $\frac{\partial \pi_i}{\partial w_i} > 0, \frac{\partial \pi_i}{\partial x_i} > 0, \frac{\partial^2 \pi_i}{\partial {w_i}^2} \leq 0, \frac{\partial^2 \pi_i}{\partial {x_i}^2} \leq 0 \ and \ \frac{\partial^2 \pi_i}{\partial x_i\, \partial w_i} > 0, \forall \ x_i,\ w_i > 0,$ i.e. the payoff function is concave in its arguments and an increase in complementary investment by a region enhances marginal productivity of water in that region.

Let $p_i \in [0,1]$ be the probability of success of region i in the case of fighting between regions, which is given by the following Contest Success Function (CSF).

$$p_i = \frac{1}{2}\left(1 + g_i - g_j\right),\ \ i, j = 1,2; i \neq j \tag{3}$$

Note that (a) $0 \leq p_i \leq 1$, since $0 \leq g_i \leq \ e_i$ and (b) $\frac{\partial p_i}{\partial g_j} < 0 < \frac{\partial p_i}{\partial g_i}\ ; \forall \ i, j = 1,2 \ \ and \ i \neq j$. It is easy to verify that the CSF given by equation (3) is translation invariant, i.e. p_i remains unchanged due to equal increase/decrease in g_i and g_j, and is well defined for all $g_i, g_j \geq 0$. In later sections of this paper we analyse the implications of more general forms of the CSF.[4]

We mention here that contest success functions (CSFs) are used to model various scenarios of conflict. There are two alternative interpretations of a CSF: (a) in terms of probability of success and (b) in terms of sharing rule. According to either of the two interpretations, the players can exert efforts to influence the expected outcome. In the first case, players exert efforts to improve their respective probabilities of win, while in the latter, they try to increase their shares in the common object. Either way, the choice of efforts is made strategically to maximize expected payoff. Only after the efforts are chosen strategically does the given CSF yield the winning probability or share deterministically. In our model, the CSF, of course, connotes a sharing rule based on the claims made by the two regions.

Higher investment in capacity-building by a region makes that region's bid for a higher share of water more credible, since higher capacity-building complementary investment leads to higher productivity of water. Competing regions may prefer to base their claims on a spree of investment activities that necessitate a higher amount of water for those investments to be productive. So, we consider claims to be an increasing function of complementary investment. For simplicity, we assume that the claim for water of region i $(= 1,2)$, c_i , is given by

$$c_i = c_i(x_i) = x_i , \; i = 1,2 \tag{4}$$

It follows that, in the event that both regions end up spending their entire endowments on complementary investments (i.e., $x_i = e_i, i = 1,2$) and are left with no fighting inputs ($g_i = 0$), the CSF (3) yields an equitable sharing rule over the contested water, i.e., $p_1 = p_2 = \frac{1}{2}$.

Regions' endowment constraints, payoff functions, contest success functions and claims are assumed to be common knowledge and there is no uncertainty involved, i.e. information is

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[4] The specific linear CSF given by equation (3) is anonymous as it yields the same share to regions for the same choice of efforts. So, such a CSF is eminently suitable for a contest where there is some semblance of mediation. This also helps explain why we get a 'cooperative' outcome as a result of the non-cooperative game. In Section 3, we extend the analysis by considering a generalized CSF $p_i(g_i, g_j)$, which is not anonymous and which takes care of a region's idiosyncrasies in determining its share.

assumed to be complete. We mention here that the above mentioned setup and functional forms, except for the form of the CSF, are in line with Ansink and Weikard (2009).[5]

We consider two alternative sequential move games between the two regions, (i) simultaneous choice of complementary investments and (ii) sequential choice of complementary investments, stages of which are as follows.

Game-1: Simultaneous choice of complementary investments

Stage 1. Competing regions choose their respective complementary investments (x_1 and x_2), independently and simultaneously.

Stage 2. Regions fight over the contested water in order to secure their respective shares.

Stage 3. Water is allocated and payoffs are realized.

Game-2: Sequential choice of complementary investments

Stage 1. Region i decides its complementary investment x_i and region j observes it; $i, j = 1, 2$ and $i \neq j$.

Stage 2. Region j decides its complementary investment x_j.

Stage 3. The two regions engage in fighting over the contested water in order to secure their respective shares.

Stage 4. Water is allocated and payoffs are realized.

We solve these two games separately by backward-induction method and characterize the sub-game perfect Nash equilibrium (SPNE). Without any loss of generality, we consider that region i decides its complementary investment before region j in the case of Game-2.

Note that once a region decides its capacity building complementary investment x_i $(= 1,\ 2)$, that region's claim for water (c_i) and fighting input (g_i) are uniquely determined through the

[5] Ansink and Weikard (2009) and several other studies consider that the CSF is given by $p_i = \frac{g_i}{g_i + g_j}$. Note that this CSF is not defined for $g_i = g_j = 0$ and, thus, cannot be considered in scenarios in which $g_i = g_j = 0$ is a possibility. While it satisfies scale-invariance property, it does not satisfy the translation-invariant property. Also note that the ratio CSF corresponds to 'ideal combat' where the player with zero effort loses all, while the difference form CSF corresponds to 'frictional combat' where the losing player can still obtain a share (see Hirshleifer, 2000).The CSF (3) is a particular characterization of the difference-form CSF in Che and Gale (2000).

claim function (4) and the endowment constraint (1) , respectively. Thus, given x_i, the claim of region i is $c_i = x_i$ and its fighting input is $g_i = e_i - x_i$, $i = 1, 2$. It follows that the amount of contested water in Stage-2 of Game-1 (Stage-3 of Game-2) is given by $(c_1 + c_2 - 1) = (x_1 + x_2 - 1)$.

Lemma 1: *When the two regions' claims over water resources of a shared river are endogenously determined, region i's capacity building complementary investment (x_i^*), claim for water (c_i^*), resource used as fighting input (g_i^*), share of water (w_i^*) and payoff (π_i^*) in the SPNE are given by.* $x_i^* = c_i^* = e_i$, $g_i^* = 0$, $w_i^* = \frac{1}{2}(1 + e_i - e_j)$ *and* $\pi_i^* = \left[\frac{e_i(1+e_i-e_j)}{2}\right]^{\alpha_i}$, *where* $e_i \in (0,1]$ *is the initial endowment of resources of region* i; $i, j = 1, 2$ *and* $i \neq j$. *This is true regardless of whether the two regions decide their respective capacity building complementary investments simultaneously (as in Game-1) or sequentially (as in Game-2).*

It is evident, from Lemma 1, that the equilibrium outcomes are not sensitive to the timing of moves by the regions. That is, regardless of whether the regions decide their complementary investments simultaneously or sequentially, the equilibrium outcomes remain the same. Further, note that the result of no expenditure on fighting by any of the regions holds true (i) for all $\alpha_i, \alpha_j \in (0,1]$ and (ii) all $e_i, e_j \in (0,1], i, j = 1,2$; i.e., regardless of whether regions are symmetric in terms of productivity and/or endowments or not. Proposition 1 summarizes these results.

Proposition 1: *When the two regions' claims over water resources of a shared river are endogenously determined, in the equilibrium no region spends any part of its endowment to fight, irrespective of*

i) whether regions are symmetric or asymmetric in terms of their resource endowment and/or productivities and

ii) whether regions decide their respective claims over water resources simultaneously or sequentially.

There are two channels through which the complementary investment affects the water share of a region. First, an increase in complementary investment by a region reduces its fighting input, which reduces its probability of success and thus, has a negative effect on the proportion of water it receives for any given claims. Second, by increasing complementary investment, a region enhances its claim over water and reduces the rival's assured water share and thus, tends to exert a favourable effect on its own share of water for any given probability of success. The positive effect of an increase in complementary investment of a region on its water share, which works via the second channel, dominates the corresponding negative effect of the first channel. As a result, a region gets a higher share of water by increasing its complementary investment. Further, note that the marginal productivity of water in a region is increasing in that region's complementary investment. Since by increasing its claims, a region not only increases its water share but also the productivity of water, it has no incentive whatsoever to invest in fighting inputs. This leads the regions to an equilibrium where both fully utilize their endowments for complementary capacity-building.

Remark 1: The equilibrium outcome of the non-cooperative game is Pareto efficient, i.e., no region can be made better off without making the other worse off. (See Appendix for proof)

Remark 2: If the two competing regions have the same amount of initial endowment and identical production functions, the non-cooperative equilibrium outcome is overall efficient, i.e. joint surplus of the two regions is maximized at the non-cooperative equilibrium. Otherwise, if regions are asymmetric, the non-cooperative equilibrium outcome may or may not be overall efficient. (See Appendix for proof)

We have observed that in the SPNE no region spends resources to fight for water, instead each region spends the entire initial endowment for complementary capacity building. If competing regions have the same amount of initial endowment ($e_1 = e_2 = e$) and the same productivity parameter ($\alpha_1 = \alpha_2 = \alpha$), and if none of the regions spends on fighting input ($g_1 = g_2 = 0$), marginal productivity of water is the same for both regions, $\frac{\partial \pi_1}{\partial w_1} = \frac{\partial \pi_2}{\partial w_2}$. Therefore, in such a scenario, joint surplus maximization calls for equal sharing of water $w_1 = w_2$, which is achieved in the non-cooperative equilibrium. However, if regions are asymmetric, i.e. if either $e_1 \neq e_2$ or $\alpha_1 \neq \alpha_2$ or both, then SPNE water allocation is given by

${w_i}^* = \frac{1}{2}\left(1 + e_i - e_j\right),\ i,j = 1,2,\ i \neq j$, which can lead to joint surplus maximization, provided that at the SPNE marginal productivity of water is the same for each region, i.e., if $\frac{\partial \pi_1}{\partial w_1} = \frac{\partial \pi_2}{\partial w_2} \Leftrightarrow \frac{\left[\frac{1}{2}(1+e_i-e_j)\right]^{\alpha_i-1}}{\left[\frac{1}{2}(1+e_j-e_i)\right]^{\alpha_j-1}} = \frac{\alpha_j {e_j}^{\alpha_j}}{\alpha_i {e_i}^{\alpha_i}}$ is satisfied at the SPNE. It follows that, if at SPNE $\frac{\partial \pi_1}{\partial w_1} \neq \frac{\partial \pi_2}{\partial w_2}$, each region can gain from engaging in trade of water. That is, while the non-cooperative equilibrium outcome does not involve any waste of resources in fighting and is Pareto efficient, regions may be able to improve further through trade in water after securing their own shares of water through non-cooperation, unless they are symmetric.

2.1 Less effective linear contest success functions

In this section we consider a more general functional form of contest success function (CSF) given by equation (5) in order to assess robustness of our results and compare the same with the findings of Skaperdas (1992).

$$p_i = \frac{1}{2} + \beta\left(g_i - g_j\right),\ i,j = 1,2; i \neq j, 0 \leq \beta \leq \frac{1}{2} \qquad (5)$$

Clearly, CSF (5) includes CSF (3) as a special case ($\beta = \frac{1}{2}$). Note that, for all $\beta \in [0, \frac{1}{2})$, the absolute values of the marginal effects of a region's fighting input (g_i) on its success probability and on its rival's success probability are less compared to those in the case of $\beta = \frac{1}{2}$.

Solving Game-1 and Game-2 by considering CSF (5), while keeping everything else same as before, we obtain the following.

Proposition 2: *Suppose that the Contest Success Function of region* $i\ (= 1,2)$ *is given by (5). Then, Proposition 1 holds true for all* $\beta \in [0, \frac{1}{2}]$.

Skaperdas (1992) analyses conflict between two players, with each having an exogenously given endowment that can be used for productive purpose or for fighting. However, unlike as in the present analysis, Skaperdas (1992), by considering that productive inputs of players are used to produce jointly and that the players fight to possess the final good, shows that it is necessary to have a 'sufficiently ineffective conflict technology' in order to obtain 'full cooperation', i.e., 'no fighting by any player', as the equilibrium outcome. Hirshleifer (1988) and Baik (1998) also demonstrate the same result[6]. If we consider the CSF (5) in Skaperdas's (1992) setup, *ceteris paribus,* the necessary condition of 'sufficiently ineffective conflict technology' turns out to be equivalent to the condition $\beta \leq \frac{1}{4}$ (see Appendix for Proof). So, Proposition 2 demonstrates that, in the present context, 'full cooperation' is the equilibrium outcome even when the conflict technology is not 'sufficiently ineffective'. In fact, we obtain a 'full cooperation' result for all possible values of $\beta \in (0, \frac{1}{2}]$.

Also, Skaperdas (1992) obtains 'full cooperation' in a setting of symmetric players, while we show that 'full cooperation' emerges as the equilibrium outcome of the non-cooperative game even when competing regions are asymmetric in terms of their initial endowments and/or productivity parameters. Further, while Skaperdas (1992) considers only a simultaneous move game, we consider both simultaneous and sequential choice of complementary investments by competing regions.

It can also be checked that (a) the SPNE outcome of non-cooperative games, Game-1 and Game-2, is Pareto efficient and (b) in case the regions are symmetric, the non-cooperative equilibrium strategies maximize the joint payoff.

3. A Generalization

Consider that region i's payoff function and CSF are given by equations (6) and (7), respectively.

$$\pi_i = f^i\big(x_i, w_i(x_i, x_j)\big) = F^i\big(x_i, x_j\big), \tag{6}$$

$$p_i = p_i\big(g_i, g_j\big), \tag{7}$$

Where (a) $f^i(0,0) = 0$, $f^i_{x_i}(x_i, w_i) > 0, f^i_{w_i}(x_i, w_i) > 0$, $f^i_{x_i x_i}(x_i, w_i) \leq 0$ and $f^i_{w_i w_i}(x_i, w_i) \leq 0$, and (b) $\frac{\partial p_i}{\partial g_i} > 0, \frac{\partial p_i}{\partial g_j} < 0, \frac{\partial p_i}{\partial g_i} = -\frac{\partial p_j}{\partial g_i}$, $p_i(1,0) \leq 1, p_i(0,1) \geq 0, p_i(0,0) \geq 0$, and $p_i\big(g_i, g_j\big) + p_j\big(g_i, g_j\big) = 1; i, j = 1, 2;\ i \neq j$.

[6] Skaperdas (1992) considers a set of axioms on the conflict technology, which can be characterized by a logistic function. Hirshleifer considers both a logistic and a ratio form CSF and finds the former to be better. Baik (1998) considers a generalized difference form CSF.

Note here that we do not assume that $p_1(0,0) = p_2(0,0)$, i.e. competing regions are not necessarily anonymous. In other words, for the same level of fighting inputs, it is possible to have different success outcomes for different regions. This allows for the possibility of unmediated contests[7] where each region uses its own contest technology. It is easy to observe that raw conflict over water sharing is better delineated by an unmediated contest than a mediated one, which corresponds better to a situation where atleast a framework for property rights exists ex-ante.

Proposition 3: *Suppose that the competing regions decide their capacity building complementary investment simultaneously and independently. Then, a strong sufficiency condition to achieve full cooperation as the equilibrium outcome of the non-cooperative game is* $\eta_i \leq \frac{e_i - x_i}{(x_i + x_j - 1)}$*, where* $\eta_i = \frac{\partial p_i}{\partial g_i} \frac{g_i}{p_i}$*;* $i, j = 1,2;\ i \neq j$.

Proposition 3 implies that, in the present context, when regions play Game-1, the 'full cooperation' result can be obtained for the class of contest success functions for which the elasticity of contest success function of a region (η_i) with respect to its own fighting input is less than or equal to $\frac{e_i - x_i}{(x_i + x_j - 1)}$. Otherwise, the 'full cooperation' result may or may not hold true.

'Full cooperation' result holds true even if $\eta_i > \frac{e_i - x_i}{(x_i + x_j - 1)}$, provided that the following condition is satisfied.

$$\frac{\partial F_i}{\partial x_i} = f_{x_i}^i + f_{w_i}^i \frac{\partial w_i}{\partial x_i} > 0 \Rightarrow \frac{f_{x_i}^i}{f_{w_i}^i} > -\frac{\partial w_i}{\partial x_i} = \left(x_i + x_j - 1\right)\frac{\partial p_i}{\partial g_i} - p_i\left(g_i, g_j\right) \quad (8)$$

We refer to Condition (8) as the *weak sufficiency condition*. It states that for full cooperation to be observed in the SPNE, marginal rate of technical substitution (MRTS) between complementary investment and water must be sufficiently high for region i to not invest in fighting inputs. In other words, the region must gain relatively more from a unit addition of infrastructure than that of water for no-fighting to be the equilibrium outcome of Game-1.

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[7] Rai and Sarin (2009) extend the axiomatization of Skaperdas (1996) to unmediated contests. According to them, the axiom of anonymity is better suited to mediated rather than unmediated contests.

Proposition 4: *When the two regions' claims over water resources of a shared river are endogenously determined through simultaneous choice of complementary investments, if one of the regions (say, i) cooperates, there will exist an equilibrium of 'partial conflict' such that the other region j invests some of its endowment in fighting if the following sufficient condition is satisfied.*

$$\left.\frac{\partial \pi_j(x_j, w_j(e_i, x_j))}{\partial x_j}\right|_{x_j=0} > 0 \text{ and } \left.\frac{\partial \pi_j(x_j, w_j(e_i, x_j))}{\partial x_j}\right|_{x_j=e_j} < 0.$$

Proposition 4 implies that there can exist a non-cooperative outcome, where one region invests entirely in complementary infrastructure, while the other invests, at least partly, in fighting. Assuming identical conflict technologies for both regions, such a 'partial conflict' equilibrium might arise in the context of upstream and downstream regions, where the marginal benefit from investment in capacity-building is sufficiently higher for the former than the latter. An upstream region stands to gain more from an extra unit of infrastructure relative to water than a downstream region. So, for identical conflict technology, there could exist an equilibrium such that the sufficient condition (8) holds for the upstream but not for the downstream region.

So far, we have demonstrated that when regions choose capacity-building complementary investments simultaneously (Game-1), in the equilibrium, either both regions cooperate or one of the two cooperates or none cooperates, depending on parametric configurations. It can be checked that the same result holds true for the sequential choice of complementary investments (Game-2) as well.

Proposition 5: *When the two regions' claims over water resources of a shared river are endogenously determined, any one of the following three possibilities emerges in the SPNE depending on relative magnitudes of marginal benefits from capacity-building investments and effectiveness of conflict technologies of competing regions, regardless of whether capacity-building investments are chosen simultaneously or sequentially.*

- *(i)* *None of the regions invests in fighting;*
- *(ii)* *Only one of the regions invests in fighting;*
- *(iii)* *Both regions invest in fighting.*

We mention here that even with different conflict effectiveness, it is possible to have partial conflict in the equilibrium. It is interesting to observe that with $\frac{\partial p_i}{\partial g_i} > \frac{\partial p_j}{\partial g_j}$, i.e., with region i having a more effective conflict technology than region j, an equilibrium entailing cooperation on part of i (and fighting on part of j) is still possible (see the proof of Proposition 5). So, a region's strategic choice eventually boils down to a calculus of the relative effectiveness of its own and its rival's productive and contest abilities.

Intuitively, while the CSF (3) can be best characterised by its anonymity condition and hence, as pertaining to a mediated contest where each region has recourse to the same contest technology, the CSF (7) delineates an unmediated contest where each region comes equipped with its own contest technology. The CSF (3), by virtue of being a mediated contest, imparts an element of negotiation to the game, leading to a Pareto efficient outcome. However, the CSF (7) allows for contest between asymmetric powers to come into full play and thus, opens up for the possibility of conflict.

4. Additional Issues

Comparative advantage of upstream: In this analysis we have assumed that none of the competing regions has any comparative advantage over the other. However, when a river flows between an upstream and a downstream region, the upstream region often enjoys a stronger position to ensure favourable outcome. To understand the implications of such comparative advantage on the equilibrium outcome, suppose that comparative advantage of the upstream region enables it to make a claim worth ϕx_i ($> x_i$) by investing x_i amount in complementary capacity building, whereas the downstream region's claim remains x_j against an investment of x_j amount in capacity building. It can be shown that 'full cooperation' can emerge in the equilibrium in such a scenario as well. To be specific, suppose that competing region's CSF is given by equation (3). Then in the SPNE, none of the regions spends resources to fight, whenever the downstream region's initial endowment (e_j) is such that it can at least match the upstream region's maximum possible claim (ϕe_i): $e_j \geq \phi e_i$, regardless of whether regions choose their complementary investments simultaneously or the upstream region moves first or the downstream region moves first (see Appendix for the proof). That is, if the downstream region's initial endowment is more than that of the upstream region and the upstream region's comparative advantage over the downstream region is not very large, 'full cooperation' emerges in the SPNE. Intuitively, existence of comparative advantage of the upstream region in terms of a relatively higher claim for its capacity-building complementary investment provides it greater incentive for capacity-building. This, in turn, induces the downstream region to try to nullify the upstream region's comparative advantage by investing in its own capacity-building, which is possible if the downstream region's endowment is sufficiently high. However, if $e_j < \phi e_i$, 'full cooperation' need not necessarily occur in the equilibrium. In case the upstream region's comparative advantage is sufficiently high, possibilities of all-out-fighting or partial conflict cannot be ruled out upfront, which remains open for future research.

Multi-player linear CSF and consistency: Skaperdas (1996) provides an axiomatic characterization of contest success functions and dismisses linear CSFs including (3), since linear CSFs do not satisfy the 'consistency' axiom: "If a nonempty subset of the players, $M \subseteq N$, were to break off from the other players and engage in a contest amongst themselves, what would be the probability of success of each player in that subset? Denote by $p_m^i(y)$ the ith player's probability of success who participates in a contest among the members of the subset M which we assume to have at least two elements. We assume this to be as follows: $p_m^i\,(y) = \frac{p^i(y)}{[\sum_{j \in M} p^j\,(y)]}\ \forall\ i \in M\ and\ \forall\ M \subseteq N$ with at least two elements." While this may appear to be a valid criticism of CSF (3), the linear CSF satisfies the following alternative definition of 'consistency'.

<u>Linear Consistency Axiom</u>: First, note that the CSF (3) can be generalized to an N-player contest as follows.

$$p_N^i = \frac{1}{N}\left[1 + (N-1)g_i - \sum_{\substack{j=1 \\ j \neq i}}^{N} g_j\right], \tag{9}$$

where p_N^i is the probability of winning of the ith player in an N-way contest. Next, define 'Linear Consistency' as follows.

$$p_n^i = \frac{1}{n}\left[1 + (n-1)p_N^i - \sum_{\substack{j=1 \\ j \neq i}}^{n} p_N^j\right] , \tag{10}$$

where n is the size of the subset of the grand coalition comprising N players. p_n^i gives the probability of winning for player i in the sub-contest amongst the n players, while p_N^i gives the probability of winning for player i in the grand contest amongst N.

It can be shown that the linear CSF (9) satisfies the linear consistency property for all $n < N$ (see Appendix for proof). Note that the above generalized linear CSF also satisfies the 'independence of irrelevant alternatives' as no outcome of any sub-contest is dependent on the inputs of players who are not part of the subset.

5. Concluding remarks

This paper tries to place conflict over shared water within the larger framework of contest literature. The primary raison d'être of the paper is to shed light on the driving mechanism behind manifest cooperation between water-sharing regions in the absence of formal cooperative arrangements. It is found that when the regions make claims to shared water on the basis of investment made prior to water sharing, in the anticipation that it would strengthen their respective positions, and when their respective shares are determined using a linear contest success function, the resulting outcome is Pareto efficient. This efficiency is obtained for both simultaneous and sequential move games and irrespective of whether the regions are symmetric or not in terms of their endowments and productivities. This result is more universal than that generally obtained in existing contest literature. One of the reasons for this is the assumption of different benefits to different regions, which is natural in the case of water sharing. Moreover, while literature like Skaperdas (1992) assume complete absence of property rights, our scheme of endogenized claims imparts a framework of property rights by granting a modicum of legitimacy to claims. This is especially true of the mediated contest which yields cooperation unequivocally. We see that the choice of contest success function doesn't make a difference, because the Skaperdas (1992) result remains even with our function. At the same time, we present a case for the linear contest function and show that it cannot be dismissed as it is done in Skaperdas (1996).

When the production and contest success functions are generalised, the efficient solution remains still, but now we find an entire spectrum of outcomes with varying degrees of cooperation and conflict, depending on the calculus of relative productive and contest efficiencies. When we allow the upstream region to enjoy some amount of comparative advantage in so much as its claims are accorded more importance, the full cooperation result is still obtained, unless said comparative advantage is sufficiently large or equivalently, the endowment of the downstream region sufficiently small.

The overall economic efficiency result obtains only under certain conditions. When regions are similar with respect to endowments and productivities, these conditions are met and the non-cooperative solution results in overall efficiency.

A natural extension of our work would be to introduce informational asymmetry and check if the possibility of cooperation still remains in the context of water sharing. The existing literature holds that conflict is very likely to surface when players overestimate their probabilities of winning (Blainey, 1973). Such concerns can be addressed by introducing private information regarding the effectiveness of conflict technology.

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Appendix

A1. Proof of Lemma 1: Let us first consider that the two competing regions decide their respective capacity building complementary investments simultaneously and independently, i.e. we begin with the Game-1. In this case, given x_1, x_2, c_1, c_2, g_1 and g_2, in stage-3 the water share for region i is given by $w_i = 1 - x_j + \frac{1}{2}(1 + g_i - g_j)(x_i + x_j - 1)$; $i,\ j = 1, 2;\ i \neq j$. Note that $1 - x_j$ is the amount of water guaranteed to region i.

It is easy to observe that in stage-2 region i's optimum choice of fighting input is given $g_i = e_i - x_i$, $i = 1, 2$. Thus, region i's share of water is given by

$$w_i = 1 - x_j + \frac{1}{2}(1 - x_i + x_j + e_i - e_j)(x_i + x_j - 1), i, j = 1,2; i \neq j. \qquad (A.1)$$

Now, in stage-1, the problem of region i can be written as follows.

$$Max_{x_i}\ \pi_i(x_i, w_i) \equiv Max_{x_i}\ \pi_i(x_i, x_j; e_i, e_j, \alpha_i)$$

$$s.t$$

$$w_i\ = 1 - x_j + \frac{1}{2}(1 - x_i + x_j + e_i - e_j)(x_i + x_j - 1), i, j = 1,2; i \neq j$$

$$\equiv\ Max_{x_i}\ z_i(x_i, x_j)$$

$$= Max_{x_i}\left[x_i\ \left\{1 - x_j + \frac{1}{2}(1 - x_i + x_j + e_i - e_j)(x_i + x_j - 1)\right\}\right]^{\alpha_i}, i, j = 1,2; i \neq j \qquad (A.2)$$

Now,

$\frac{\partial z_i(x_i,x_j)}{\partial x_i} = \left(w_i + x_i \frac{\partial w_i(x_i,x_j)}{\partial x_i}\right)(x_i\ \ w_i(x_i, x_j))^{\alpha_i - 1}\ \alpha_i$, where

$$\frac{\partial w_i(x_i,x_j)}{\partial x_i} = \frac{1}{2}\left[(1 - x_i + x_j + e_i - e_j) - (x_i + x_j - 1)\right] = \frac{1}{2}\left[2 - 2x_i + e_i - e_j\right]$$

$= \frac{1}{2}\left[(1 - x_i) + (1 - e_j) + (e_i - x_i)\right] \geq 0$, since $e_i, e_j\ \in (0,1]$ and $x_i \leq e_i$, which is satisfied with equality if $x_i = e_i = 1$ and $e_j = 1$.

Therefore, $\frac{\partial z_i(x_i,x_j)}{\partial x_i} > 0\ \forall\ x_i\ \epsilon\ [0, e_i], i, j = 1,2; i \neq j.$

Hence, the solution to problem (A.2) is $x_i{}^* = e_i$, which implies $g_i{}^* = 0$. Therefore, the equilibrium water share and payoff of region i are, respectively, given by $w_i{}^* = \frac{1}{2}(1 + e_i - e_j)$ and $\pi_i{}^* = [\frac{e_i}{2}(1 + e_i - e_j)]^{\alpha_i}$, $i, j = 1,2; i \neq j$.

Game-2:

The problem of the second mover region j in stage 2 can be written as follows.

$$Max_{x_j} \ \pi_j \ (x_j, w_j(x_i, x_j)); i, j = 1,2; i \neq j, \qquad (A.3)$$

where $\pi_j \left(x_j, w_j(x_i, x_j)\right) = [x_j \left\{1 - x_i + \frac{1}{2}(1 - x_j + x_i + e_j - e_i)(x_i + x_j - 1)\right\}]^{\alpha_j}$. Now, it is easy to check that, for any given x_i, $\frac{d\pi_j}{dx_j} = \frac{\partial \pi_j}{\partial x_j} + \frac{\partial \pi_j}{\partial w_j}\frac{\partial w_j}{\partial x_j} > 0$, since (a) $\frac{\partial \pi_j}{\partial x_j} > 0$ and $\frac{\partial \pi_j}{\partial w_j} > 0$ from (2) and (b) $\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[(1 - x_j) + (1 - e_i) + (e_j - x_j)\right] \geq 0$ since $0 \leq x_j \leq e_j \leq 1; i, j = 1,2; i \neq j$. It implies that $x_j = e_j$is the solution of problem (A.3).

Now, in stage 1, player i's problem can be written as follows.

$$Max_{x_i} \pi_i = \left[x_i w_i(x_i, x_j)\right]^{\alpha_i}, i, j = 1,2; i \neq j, \qquad (A.4)$$

where, $w_i(x_i, x_j) = \left\{1 - e_j + \frac{1}{2}(1 - x_i + e_i)(x_i + e_j - 1)\right\}$.

Now,

$\frac{\partial w_i(x_i, x_j)}{\partial x_i} = \frac{1}{2}\left[(1 - x_i + x_j + e_i - e_j) - (x_i + x_j - 1)\right] = \frac{1}{2}\left[2 - 2x_i + e_i - e_j\right] =$
$\frac{1}{2}\left[(1 - x_i) + (1 - e_j) + (e_i - x_i)\right] \geq 0$, since $e_i, e_j \in (0,1]$ and $x_i \leq e_i$. It follows that $x_i = e_i$ is the solution of problem (8).

Hence, the sub-game perfect Nash equilibrium (SPNE) of the sequential move game and corresponding payoffs are as in Lemma 2. [QED]

A2. Proof of Remark 1: Pareto Efficiency of the SPNE

Definition (Pareto Efficiency): An outcome pair $\{(c_i{}^*, g_i{}^*), (c_j{}^*, g_j{}^*)\}$ is Pareto efficient if there exists no outcome pair $\{(c_i, g_i), (c_j, g_j)\} \neq \{(c_i{}^*, g_i{}^*), (c_j{}^*, g_j{}^*)\}$ such that $\pi_i(c_i, w_i(c_i, g_i, c_j, g_j)) \geq \pi_i{}^*(c_i{}^*, w_i{}^*(c_i{}^*, g_i{}^*, c_j{}^*, g_j{}^*))$ and $\pi_j(c_j, w_j(c_i, g_i, c_j, g_j)) \geq \pi_j{}^*(c_j{}^*, w_j{}^*(c_i{}^*, g_i{}^*, c_j{}^*, g_j{}^*))$, with strict inequality for at least one of i, j; $i, j = 1,2; i \neq j$.

Now, to prove that the non-cooperative outcome is Pareto efficient, we need to show that it lies on the payoff possibility frontier for negotiated outcomes or is, in fact, the same as the cooperative outcome. The payoff possibility frontier maps all Pareto efficient combinations of payoffs. We, therefore, first characterize the payoff possibility frontier. In order to do so, we consider that, given the claims, the two regions engage in Nash bargaining over water sharing. In the Nash bargaining, the threat points of the two regions are given by their respective payoffs in the equilibrium of the non-cooperative game, which are $\pi_i{}^* = [\frac{e_i}{2}(1 + e_i - e_j)]^{\alpha_i}$ and $\pi_j{}^* = [\frac{e_j}{2}(1 + e_j - e_i)]^{\alpha_j}$. Also note that, in the equilibrium of the non-cooperative play, we have $\left(x_i{}^*, x_j{}^*\right) = (e_i, e_j)$.

The bargaining problem can be written as follows.

$$Max_{w_i} Z = [(x_i w_i)^{\alpha_i} - \pi_i{}^*]\left[\{x_j(1 - w_i)\}^{\alpha_j} - \pi_j{}^*\right] \qquad (A.5)$$

Let us denote the equilibrium payoffs under Nash bargaining by $\pi_i = (x_i w_i)^{\alpha_i}\, and\, \pi_j = (x_j w_j)^{\alpha_j}$. Then, the payoff possibility frontier is given by

$$\pi_i = [\frac{x_i(x_j - \pi_j{}^{\frac{1}{\alpha_j}})}{x_j}]^{\alpha_i} \qquad (A.6)$$

It is straightforward to observe that the payoff possibility frontier given by (A.6) is satisfied at $\left(\pi_i, \pi_j\right) = \left(\pi_i{}^*, \pi_j{}^*\right)$ and $\left(x_i, x_j\right) = \left(x_i{}^*, x_j{}^*\right)$. So, the non-cooperative equilibrium point lies on the payoff possibility frontier and hence, is Pareto efficient. We can also show that the non-cooperative solution happens to be indeed the same as the Nash bargaining outcome.

The First Order Condition (FOC) for the Nash bargaining exercise yields the following:

$$\alpha_i x_i{}^{\alpha_i} w_i{}^{\alpha_i - 1}\left[\{x_j(1 - w_i)\}^{\alpha_j} - \pi_j{}^*\right] - \alpha_j x_j{}^{\alpha_j}(1 - w_i)^{\alpha_j - 1}[(x_i w_i)^{\alpha_i} - \pi_i{}^*] = 0 \qquad (A.7)$$

It is evident that the above equation is satisfied when $\{x_j(1-w_i)\}^{\alpha_j} = \pi_j^*$ and $(x_i w_i)^{\alpha_i} = \pi_i^*, i.e.$, the non-cooperative outcome satisfies the above equation. In other words, the non-cooperative outcome is the same as the cooperative outcome. [QED]

A3. Proof of Remark 2: Overall Efficiency

Definition (Overall Efficiency); An outcome pair $\{(c_i^*, g_i^*), (c_j^*, g_j^*)\}$ is said to be overall efficient, if it maximizes the joint payoff, i.e. if the following is true.

$$(c_i^*, g_i^*, c_j^*, g_j^*) \in \underset{c_i, g_i, c_j, g_j}{Argmax} \left[\pi_i\left(c_i, w_i(c_i, g_i, c_j, g_j)\right) + \pi_j\left(c_j, w_j(c_i, g_i, c_j, g_j)\right)\right].$$

Now, for any given x_i, x_j, the problem of joint payoff maximization can be written as follows:

$$Max_{\{w_i\}} Z = (x_i w_i)^{\alpha_i} + \left(x_j(1-w_i)\right)^{\alpha_j}, i,j = 1,2; i \neq j \qquad (A.8)$$

The FOC of the above problem is as follows:

$$\alpha_i x_i^{\alpha_i} w_i^{\alpha_i - 1} - \alpha_j x_j^{\alpha_j}(1-w_i)^{\alpha_j - 1} = 0, i,j = 1,2; i \neq j \qquad (A.9)$$

The second-order condition is satisfied since $\frac{\partial^2 Z}{\partial w_i^2} \leq 0$, for all $\alpha_i, \alpha_j \in (0,1]$.

Further, note that $\frac{\partial Z}{\partial x_i} > 0$ and $\frac{\partial Z}{\partial x_j} > 0$ for all $\alpha_i, \alpha_j \in (0,1]$ and $w_i > 0, i,j = 1,2; i \neq j$. It implies that $x_i = e_i, x_j = e_j$ and (A.9) together determine the equilibrium under joint surplus maximization.

Now, in the equilibrium of the non-cooperative game, we have

$x_i^* = e_i, w_i^* = \frac{1}{2}(1 + e_i - e_j), i,j = 1,2. i \neq j$ and it satisfies condition (14), if

$$\alpha_i e_i^{\alpha_i}\left[\frac{1}{2}(1+e_i-e_j)\right]^{\alpha_i - 1} - \alpha_j e_j^{\alpha_j}\left[\frac{1}{2}(1+e_j-e_i)\right]^{\alpha_j - 1} = 0$$

$$\Rightarrow \frac{\left[\frac{1}{2}(1+e_i-e_j)\right]^{\alpha_i - 1}}{\left[\frac{1}{2}(1+e_j-e_i)\right]^{\alpha_j - 1}} = \frac{\alpha_j e_j^{\alpha_j}}{\alpha_i e_i^{\alpha_i}} \qquad (A.10)$$

It is straightforward to observe that a sufficient condition for (A.10) to be satisfied is as follows. $\alpha_i = \alpha_j\ and\ e_i = e_j$. [QED]

A4. Proof of Proposition 2: We first show that Proposition 1 goes through when CSF (5) is considered. We show it for the simultaneous and sequential move cases separately.

Game-1: Simultaneous moves

We can write region *i(1,2)'s* problem in stage-1 as follows.

$$Max_{x_i}\, \pi_i\left(x_i, w_i(x_i, x_j)\right)$$

$$\equiv Max_{x_i}\, v_i(x_i, x_j) = \left[x_i\ \left\{1 - x_j + \left[\tfrac{1}{2} + \beta\left(e_i - e_j + x_j - x_i\right)\right]\left(x_i + x_j - 1\right)\right\}\right]^{\alpha_i}, i, j = 1,2; i \neq j \quad (A.11)$$

To prove that the regions do not invest in fighting inputs, it is sufficient to prove that $\frac{\partial w_i(x_i,x_j)}{\partial x_i} \geq 0\ \forall\ \beta \in [0, \frac{1}{2})$. If $\frac{\partial w_i(x_i,x_j)}{\partial x_i} \geq 0$, then $\frac{\partial v_i(x_i,x_j)}{\partial x_i} > 0$.

$$\frac{\partial w_i(x_i, x_j)}{\partial x_i} = \frac{1}{2} + \beta\left(e_i - e_j + x_j - x_i\right) - \beta\left(x_i + x_j - 1\right) = \frac{1}{2} + \beta\left(e_i - e_j - 2x_i + 1\right)$$

So, we have

$$\frac{\partial w_i(x_i,x_j)}{\partial x_i} \geq 0$$

$$\Leftrightarrow \frac{1}{2} + \beta\left(e_i - e_j - 2x_i + 1\right) \geq 0 \quad (A.12)$$

Now, lowest possible value of $e_i - e_j - 2x_i + 1 = e_i - e_j - 2e_i + 1 = 1 - e_i - e_j$.

If $1 - e_i - e_j \geq 0$, then (A.12) is satisfied.

If $1 - e_i - e_j < 0$, then for (A.12) to hold, $\frac{1}{2} - \beta\left(e_i + e_j - 1\right) \geq 0 \Rightarrow \beta\left(e_i + e_j - 1\right) \leq \frac{1}{2}$.

Since maximum value of $e_i + e_j - 1 = 1$ $(at\ e_i = e_j = 1)$, (A.12) goes through for all $\beta \leq \frac{1}{2}$.

Game-2: Sequential moves

We can write the problem of the second mover region j (1,2) in stage 2 as follows:

$$Max_{x_j}\ \pi_j\ \left(x_j, w_j(x_i, x_j)\right)$$

$$\equiv Max_{x_j}\ \left[x_j\ \left\{1 - x_i + \left[\frac{1}{2} + \beta\left(e_j - e_i + x_i - x_j\right)\right]\left(x_i + x_j - 1\right)\right\}\right]^{\alpha_j} \qquad (A.13)$$

We again use the sufficiency condition $\frac{\partial w_j(x_i,x_j)}{\partial x_j} \geq 0\ \forall\ \beta \in [0,\frac{1}{2})$ to prove that the second region never invests in fighting inputs. We need to prove the following:

$$\frac{\partial w_j(x_i,x_j)}{\partial x_j} = \frac{1}{2} + \beta\left(e_j - e_i + x_i - x_j\right) - \beta\left(x_i + x_j - 1\right) = \frac{1}{2} + \beta\left(e_j - e_i - 2x_j + 1\right) \geq 0.$$

Following the preceding proof, it is clear that $\frac{1}{2} + \beta\left(e_j - e_i - 2x_j + 1\right) \geq 0\ \forall\ \beta \in [0,\frac{1}{2})$. So, $x_j = e_j$ is the solution to problem (A.13).

Player i's problem in the first stage can now be written as follows:

$$Max_{x_i}\,\pi_i = \left[x_i\ \left\{1 - e_j + [\tfrac{1}{2} + \beta(e_i - x_i)]\left(x_i + e_j - 1\right)\right\}\right]^{\alpha_i} \qquad (A.14)$$

Again, we require the following sufficiency condition:

$$\frac{\partial w_i(x_i,x_j)}{\partial x_i} = \frac{1}{2} + \beta(e_i - x_i) - \beta\left(x_i + e_j - 1\right) = \frac{1}{2} + \beta\left(e_i - e_j - 2x_i + 1\right) \geq 0,$$

which always goes through for $\beta \in [0,\frac{1}{2})$, by virtue of the preceding arguments.

So, $\{c_i^* = e_i\,, g_i^* = 0\}$ remains the solution to both simultaneous and sequential move games. Since the equilibrium outcome remains unchanged at $\{c_i^* = e_i\,, g_i^* = 0\}$, it is evident that Remark 1 and Remark 2 also remain valid. [QED]

A5. Proof of the necessary condition of 'sufficiently ineffective conflict technology' as defined in Skaperdas (1992)

We now prove that the *Full Cooperation* result of Skaperdas (1992) (Proposition 1a(i)) doesn't go through entirely if the CSF (22) is considered. For this purpose, consider the model set-up to be exactly same as in Skaperdas (1992), except that the conflict technology there is replaced by the CSF (22). Then, we have the following.

$$V^1(g_1, g_2) = [\tfrac{1}{2} + \beta(g_1 - g_2)]\ C(1 - g_1, 1 - g_2)$$

$$V^2(g_1, g_2) = \left[\tfrac{1}{2} + \beta(g_2 - g_1)\right]\ C(1 - g_1, 1 - g_2) \qquad (A.15)$$

It is evident that

$\frac{\partial^2 V^1(g_1,g_2)}{\partial {g_1}^2} = V^1_{11} = -2\beta C_1 + \left[\frac{1}{2} + \beta(g_1 - g_2)\right] C_{11} < 0$ and $\frac{\partial^2 V^2(g_1,g_2)}{\partial {g_2}^2} = V^2_{22} = -2\beta C_2 + [\frac{1}{2} + \beta(g_2 - g_1)]\ C_{22} < 0\ \forall\ g_1, g_2,$

where $C_i = \frac{\partial C}{\partial x_i} > 0$, $C_{ii} = \frac{\partial^2 C}{\partial {x_i}^2} < 0$; $x_i = 1 - g_i$.

To prove that $(g_1, g_2) = (0,0)$ is an equilibrium, the following is necessary and sufficient.

$$\frac{\partial V^1(g_1,g_2)}{\partial g_1} = V^1_1(0,0) \leq 0\ and\ \frac{\partial V^2(g_1,g_2)}{\partial g_2} = V^2_2(0,0) \leq 0 \qquad (A.16)$$

$$V^1_1(\ g_1, g_2) = \beta C - [\tfrac{1}{2} + \beta(g_1 - g_2)]C_1$$

$$V^2_2(g_1, g_2) = \beta C - [\tfrac{1}{2} + \beta(g_2 - g_1)]C_2 \qquad (A.17)$$

Combining (A.16) and (A.17), we get

$$V^1_1(0,0) = \beta C(1,1) - [\tfrac{1}{2} + \beta(g_1 - g_2)]C_1(1,1) \leq 0$$

$$V^2_2(0,0) = \beta C(1,1) - \left[\tfrac{1}{2} + \beta(g_2 - g_1)\right] C_2(1,1) \leq 0 \qquad (A.18)$$

Since $C(\ .\ ,\ \ .)$ is assumed to be homogeneous of degree 1, we have

$$C(1,1) = C_1(1,1) + C_2(1,1)$$

So, we obtain $\left(\beta - \frac{1}{2}\right) C_1(1,1) + \beta C_2(1,1) \leq 0$ and $\left(\beta - \frac{1}{2}\right) C_2(1,1) + \beta C_1(1,1) \leq 0$

or, $\qquad \frac{2\beta}{1-2\beta} \leq \frac{C_1(1,1)}{C_2(1,1)} \leq \frac{1-2\beta}{2\beta}$.

So, full cooperation in Skaperdas (1992) obtains only when $\frac{2\beta}{1-2\beta} \leq \frac{1-2\beta}{2\beta}$ or, $\beta \leq \frac{1}{4}$.

A6. Proof of Proposition 3: We have the following from (6):

$$\frac{\partial F_i}{\partial x_i} = f^i_{x_i} + f^i_{w_i} \frac{\partial w_i}{\partial x_i} \qquad (A.19)$$

To prove that $\frac{\partial F_i}{\partial x_i} > 0$, it is sufficient to prove that $\frac{\partial w_i}{\partial x_i} \geq 0$ since $f^i_{x_i}(x_i, w_i) > 0$ and $f^i_{w_i}(x_i, w_i) > 0$ by assumption. We have the following:

$$w_i = 1 - x_j + p_i(g_i, g_j)\,[x_i + x_j - 1] \equiv 1 - x_j + p_i(e_i - x_i, e_j - x_j)\,[x_i + x_j - 1]$$

$$\Rightarrow \frac{\partial w_i}{\partial x_i} = p_i(e_i - x_i, e_j - x_j) + (x_i + x_j - 1)\frac{\partial p_i}{\partial(e_i - x_i)}\,(-1)$$

$$= p_i(g_i, g_j) + (x_i + x_j - 1)\frac{\partial p_i}{\partial g_i}\,(-1) \geq 0$$

$$\Leftrightarrow \frac{\partial p_i(g_i, g_j)}{\partial g_i}\,\frac{1}{p_i(g_i, g_j)} \leq \frac{1}{(x_i + x_j - 1)}$$

$$\equiv \frac{\partial p_i(g_i, g_j)}{\partial g_i}\,\frac{g_i}{p_i(g_i, g_j)} = \eta_i \leq \frac{e_i - x_i}{(x_i + x_j - 1)} \qquad \text{[QED]}$$

A7. Proof of Proposition 4: Let either of the sufficiency conditions for 'full cooperation' hold for region i. So, region i invests entirely in complementary infrastructure, i.e., $x_i = e_i$. Now, region j's reaction function is given by $x_j(x_i)$, which, in equilibrium, yields $x_j(e_i)$.

To prove that region j invests some amount in fighting, we need to prove $x_j(e_i) < e_j$, i.e., there is an interior solution.

Since $f^i_{x_i x_i}(x_i, w_i) \leq 0$ by assumption, for an interior solution, the following is sufficient:

$\left.\frac{\partial \pi_j(x_j, w_j(e_i, x_j))}{\partial x_j}\right|_{x_j=0} > 0$ and $\left.\frac{\partial \pi_j(x_j, w_j(e_i, x_j))}{\partial x_j}\right|_{x_j=e_j} < 0.$ [QED]

A8. Proof of Proposition 5: Sequential moves:

Let region i move first and region j move second.

In the second stage, region j's problem is to choose

$$x_j \in argmax_{x_j}\left[\pi_j\left(x_j, w_j\right)\right] \qquad (A.20)$$

Now, $x_j = e_j$, if $\eta_j \leq \frac{e_j - x_j}{(x_i + x_j - 1)}$ or condition (8) holds.

$(A.21)$

$$x_j < e_j \text{ if } \left.\frac{\partial\pi_j(x_j, w_j(x_i, x_j))}{\partial x_j}\right|_{x_j=0} > 0 \text{ and } \left.\frac{\partial\pi_j(x_j, w_j(x_i, x_j))}{\partial x_j}\right|_{x_j=e_j} < 0 \qquad (A.22)$$

In the first stage, region i's problem is to choose

$$x_i \in argmax_{x_i}\left[\pi_i\left(x_i, w_i\left(x_i, x_j(x_i)\right)\right)\right] \qquad (A.23)$$

$$x_i = e_i \text{if } \eta_i \leq \frac{e_i - x_i}{(x_i + x_j - 1)} \text{ or condition (8) holds.} \qquad (A.24)$$

$$x_i < e_i \text{ if } \left.\frac{\partial\pi_i\left(x_i, w_i\left(x_i, x_j(x_i)\right)\right)}{\partial x_i}\right|_{x_i=0} > 0 \text{ and } \left.\frac{\partial\pi_i\left(x_i, w_i\left(x_i, x_j(x_i)\right)\right)}{\partial x_i}\right|_{x_i=e_i} < 0 \qquad (A.25)$$

There will be

(i) full cooperation if $(A.21)$ and $(A.24)$ hold together;

(ii) partial conflict if either $(A.21)$ and $(A.25)$ or $(A.22)$ and $(A.24)$ hold together;

(iii) conflict if $(A.22)$ and $(A.25)$ hold together.

It is straightaway observed that such an array of equilibria will also exist for the simultaneous move game.
[QED]

A9. Proof of the Sufficient Condition for Full Cooperation under Comparative Advantage of Upstream Region

To show that proposition 1 holds, we prove it for both simultaneous and sequential cases.

Sequential Moves:

(I) Let the upstream region i move first.

We can write the problem of the downstream region j as follows:

$$Max_{x_j} \;\; \pi_j \; (x_j, w_j(x_i, x_j)); i, j = 1,2; i \neq j \qquad (A.26)$$

where $\pi_j \; \left(x_j, w_j\left(x_i, x_j\right)\right) = [x_j \; \left\{1 - \phi x_i + \frac{1}{2}\left(1 - x_j + x_i + e_j - e_i\right)\left(\phi x_i + x_j - 1\right)\right\}]^{\alpha_j}$

In order to prove that region j will not invest in fighting input, it is sufficient to prove that $\frac{\partial w_j}{\partial x_j} \geq 0$ for $x_j \; \in [0, e_j]$. We have

$$\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[\left(2 - 2x_j\right) + \left(e_j - e_i\right) - x_i(\phi - 1)\right]$$

$\frac{\partial w_j}{\partial x_j}$ is minimum at $x_i = e_i$; so if $\frac{\partial w_j}{\partial x_j} \geq 0$ at $x_i = e_i$, then $\frac{\partial w_j}{\partial x_j} \geq 0 \; \forall x_i < e_i$.

Now, at $x_i = e_i$, $\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[\left(2 - 2x_j\right) + \left(e_j - \phi e_i\right)\right] \geq 0 \iff \phi \leq \frac{e_j}{e_i} \Rightarrow e_j \geq \phi e_i$.

So, the downstream region j chooses $x_j = e_j$ if its endowment is sufficiently large, i.e., greater than or equal to ϕe_i.

The problem of the upstream region is as follows:

$$Max_{x_i} \, \pi_i = \left[x_i w_i(x_i, e_j)\right]^{\alpha_i}, i, j = 1,2; i \neq j \qquad (A.27)$$

where $w_i(x_i, e_j) = \left\{1 - e_j + \frac{1}{2}(1 - x_i + e_i)\left(\phi x_i + e_j - 1\right)\right\}$.

Now,

$$\frac{\partial w_i}{\partial x_i} = \frac{1}{2}\left[\left(1 - e_j\right) + \phi(1 - x_i) + \phi(e_i - x_i)\right] \geq 0 \; \forall \; x_i \in [0, e_i]$$

So, the first mover upstream region indeed chooses $x_i = e_i$ in response to $x_j = e_j$. So, for a small enough ϕ ($\leq \frac{e_j}{e_i}$), which materializes when e_j is greater than or equal to ϕe_i, 'full cooperation' obtains when the upstream region moves first.

(II) Let the downstream region j move first.

We can write the problem of the second mover upstream region i as follows:

$$Max_{x_i} \ \pi_i \ (x_i, w_i(x_i, x_j)); i, j = 1,2; i \neq j \tag{A.28}$$

where $\pi_i \ \left(x_i, w_i(x_i, x_j)\right) = [x_i \ \left\{1 - x_j + \frac{1}{2}(1 - x_i + x_j + e_i - e_j)(\phi x_i + x_j - 1)\right\}]^{\alpha_i}$

In order to prove that the upstream region will not invest in fighting input, it is sufficient to prove that $\frac{\partial w_i}{\partial x_i} \geq 0$ for $x_i \ \in [0, e_i]$. We have

$$\frac{\partial w_i}{\partial x_i} = \frac{1}{2}\left[(1 - x_j) + \phi(1 - e_j) + \phi(e_i - x_i) + \phi(x_j - x_i)\right] \geq 0 \text{ if } \ x_j \geq x_i.$$

So, upstream region chooses $x_i = e_i$ if $x_j \geq x_i$.

The problem of the first mover downstream region is as follows:

$$Max_{x_j} \pi_j = \left[x_j w_j(e_i, x_j)\right]^{\alpha_j}, i, j = 1,2; i \neq j \tag{A.29}$$

where $w_j(e_i, x_j) = \ 1 - \phi e_i + \frac{1}{2}(1 - x_j + e_j)(\phi e_i + x_j - 1)$.

Now,

$$\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[(2 - 2x_j) + (e_j - \phi e_i)\right] \geq 0 \ \Leftrightarrow \ \phi \leq \frac{e_j}{e_i} \Rightarrow e_j \geq \ \phi e_i$$

Also, if $e_j \geq \ \phi e_i \ \Rightarrow \ e_j \geq \ e_i \Rightarrow e_j \geq \ x_i$, which, in a 'full cooperation' equilibrium, , implies $x_j \geq \ x_i$.

So, for a small enough comparative advantage enjoyed by the upstream region, both the upstream and the downstream regions invest entirely in complementary investment, regardless of who moves first.

Simultaneous moves:

We can write the problem of the downstream region j as follows.

$$Max_{x_j}\ \pi_j\ (x_j, w_j(x_i, x_j)); i, j = 1,2; i \neq j \qquad (A.30)$$

where $\pi_j\ \left(x_j, w_j\left(x_i, x_j\right)\right) = [x_j\ \left\{1 - \phi x_i + \frac{1}{2}\left(1 - x_j + x_i + e_j - e_i\right)\left(\phi x_i + x_j - 1\right)\right\}]^{\alpha_j}$

In order to prove that the downstream region will not invest in fighting input, it is sufficient to prove that $\frac{\partial w_j}{\partial x_j} \geq 0$ for $x_j\ \in [0, e_j]$. We have

$$\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[\left(2 - 2x_j\right) + \left(e_j - e_i\right) - x_i(\phi - 1)\right]$$

$\frac{\partial w_j}{\partial x_j}$ is minimum at $x_i = e_i$. If $\frac{\partial w_j}{\partial x_j} \geq 0$ at $x_i = e_i$, then $\frac{\partial w_j}{\partial x_j} \geq 0\ \forall\ x_i < e_i$.

Now, we have $\frac{\partial w_j}{\partial x_j} = \frac{1}{2}\left[\left(2 - 2x_j\right) + \left(e_j - \phi e_i\right)\right] \geq 0\ \forall\ x_j\ \in \left[0, e_j\right]$

if $\phi \leq \frac{e_j}{e_i} \Rightarrow e_j \geq \phi e_i$.

So, the downstream region j chooses $x_j = e_j$ if its endowment is sufficiently large, i.e., greater than or equal to ϕe_i.

The problem of the upstream region i is as follows:

$$Max_{x_i}\, \pi_i = \left[x_i w_i(x_i, x_j)\right]^{\alpha_i}, i, j = 1,2; i \neq j \qquad (\mathrm{A.31})$$

where $w_i(x_i, x_j) = \left\{1 - x_j + \frac{1}{2}\left(1 + e_i - e_j + x_j - x_i\right)\left(\phi x_i + x_j - 1\right)\right\}$.

Now, $\frac{\partial w_i}{\partial x_i} = \frac{1}{2}\left[\left(1 - x_j\right) + \phi\left(1 - e_j\right) + \phi(e_i - x_i) + \phi\left(x_j - x_i\right)\right] \geq 0$ if $x_j \geq x_i$.

So, the upstream region also chooses $x_i = e_i$ for $x_j \geq x_i$, which, in equilibrium, is always satisfied for $e_j > e_i$.

It is easily observed that propositions 2 and 3 also hold unless the comparative advantage enjoyed by the upstream region is sufficiently large. [QED]

A 10. Proof of Linear Consistency

The probability of the ith player winning in an n-way sub-contest is given by the CSF (9) as follows:

$$p_n^i = \frac{1}{n}\left[1 + (n-1)g_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} g_j\right] = \frac{1}{n}\left[1 + ng_i - \sum_{i=1}^{n} g_i\right] \qquad (A.32)$$

We have to prove that the probability of winning in the sub-contest, as given by (A.32), is the same as that in (10).

From (10), we have

$$p_n^i = \frac{1}{n}\left[1 + (n-1)p_N^i - \sum_{\substack{j=1 \\ j\neq i}}^{n} p_N^j\right] = \frac{1}{n}\left[1 + np_N^i - \sum_{i=1}^{n} p_N^i\right]$$

$$= \frac{1}{n}\left[1 + n\left\{\frac{1}{N}\left(1 + Ng_i - \sum_{i=1}^{N} g_i\right)\right\} - \left\{\frac{1}{N}\left(n + N\sum_{i=1}^{n} g_i - n\sum_{i=1}^{N} g_i\right)\right\}\right]$$

$$= \frac{1}{n}\left[1 + ng_i - \left(\sum_{i=1}^{N} g_i\right)\left(\frac{n}{N} - \frac{n}{N}\right) - \sum_{i=1}^{n} g_i\right]$$

$$= \frac{1}{n}\left[1 + ng_i - \sum_{i=1}^{n} g_i\right]$$

$$= \frac{1}{n}\left[1 + (n-1)g_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} g_j\right] \qquad \text{[QED]}$$