WP-2021-020

Strategic Inattention and Divisionalization in Duopoly

Promit Kanti Chaudhuri

Indira Gandhi Institute of Development Research, Mumbai
July 2021

Strategic Inattention and Divisionalization in Duopoly

Promit Kanti Chaudhuri

Email(corresponding author): promit@igidr.ac.in

Abstract

In this paper, a differentiated product economy is modeled where firms strategically set up autonomous rival divisions and the divisions play the quantity competition game `a la Cournot or by means of monopolistic competition, where the divisions are unaware of the impact of their output either on the firm's total output or on the total industry output. This case of divisions being unaware of the impact of their outputs on the firm's aggregate output or on the industry total output is termed as 'Strategic Inattention'. The incentive to divisionalize still remains within the firms even in the case of the 'Strategic Inattention', but the incentive is lower than the case of normal Cournot competition. Next in a duopoly, the firms play a three stage game. In the first stage, the firms decide whether to let their divisions utilize or ignore the information on the impact of their individual output on the firm's total output or industry total output. In the second stage the firms strategically decide on the number of divisions and in the final stage the divisions compete against each other in terms of quantity. It is seen that one firm deciding to be inattentive to the information available and the other firm using that information, is the equilibrium outcome. Thus inattentive and attentive firms coexist in a Subgame Perfect Nash Equilibrium. This result is in sharp contrast to the findings of Cellini et al. (2020).

Keywords: Divisionalization, Information, Monopolistic Competition, Oligopoly, Strategic Interaction

JEL Code: D43;L11;L13

Acknowledgements:

I am deeply grateful to Rupayan Pal for his invaluable insights and suggestions for this piece of work.

Strategic Inattention and Divisionalization in Duopoly

Promit Kanti Chaudhuri

Indira Gandhi Institute of Development Research (IGIDR)

Film City Road, Santosh Nagar, Goregaon (E) - Mumbai, 400065, India.

Email: promit@igidr.ac.in, promitkanti@gmail.com

Abstract

In this paper, a differentiated product economy is modeled where firms strategically set up autonomous rival divisions and the divisions play the quantity competition game à la Cournot or by means of monopolistic competition, where the divisions are unaware of the impact of their output either on the firm's total output or on the total industry output. This case of divisions being unaware of the impact of their outputs on the firm's aggregate output or on the industry total output is termed as 'Strategic Inattention'. The incentive to divisionalize still remains within the firms even in the case of the 'Strategic Inattention', but the incentive is lower than the case of normal Cournot competition. Next in a duopoly, the firms play a three stage game. In the first stage, the firms decide whether to let their divisions utilize or ignore the information on the impact of their individual output on the firm's total output or industry total output. In the second stage the firms strategically decide on the number of divisions and in the final stage the divisions compete against each other in terms of quantity. It is seen that one firm deciding to be inattentive to the information available and the other firm using that information, is the equilibrium outcome. Thus inattentive and attentive firms coexist in a Subgame Perfect Nash Equilibrium. This result is in sharp contrast to the findings of Cellini et al. (2020).

Keywords: Divisionalization, Information, Monopolistic Competition, Oligopoly, Strategic Interaction

JEL Classification: D43;L11;L13

Corresponding Address: Indira Gandhi Institute of Development Research (IGIDR), Film City Road, Santosh Nagar, Goregaon (East), Mumbai 400065, India. E-mail: promit@igidr.ac.in, promitkanti@gmail.com. Telephone: +91-8777466370

I am deeply grateful to Rupayan Pal for his invaluable insights and suggestions for this piece of work.

1 Introduction

It is often argued in the literature of oligopolistic competition that in markets with similar products, a decrease in competition benefits the firms by increasing their profits. However in reality many firms often set up independently managed rival divisions (e.g. the case of General Motors, Sloan (1990)) producing similar products, thus increasing competition. There have been multiple explanations for this paradoxical phenomenon, starting from Williamson (1973) arguing that, for large organizations divisionalization helps in solving the incentive problems arising from moral hazard, to explanations like, multiple divisions help in serving heterogeneous customers differing in tastes or locations. But neither of them explains why the mother firm lets the divisions to compete with each other rather than cooperate, in production.

Later works by Corchón (1991) and Polasky (1992), tried to give more convincing explanations of this rather puzzling phenomenon by setting up a two stage game in a duopoly market, where in the first stage firms decide on the number of autonomous divisions and on the second stage the divisions play quantity competition game independently à la Cournot. Their results showed that there lies an incentive for the firms to divisionalize but their Nash Equilibrium was not finite. Baye et al. (1996) tackled this problem of nonexistence of Nash Equilibrium by assuming that divisionalization is costly. Further developments by Yuan (1999) suggested a model with differentiated product markets, where even if with divisionalization being cost-less ensured an interior Subgame Perfect Nash Equilibrium (SPNE). A firm's incentive to divisionalize is reduced if products are differentiated, thus a heterogeneous product market alone is enough to ensure an interior SPNE.

The quantity competition played à la Cournot in the stage game, described earlier, assumes that the divisions choose their output levels keeping in mind the impact of their individual outputs on the industry output as well as their mother firm's output. Where as in a monopolistic competition the divisions take either the mother firm's output or the industry output as given. Dixit and Stiglitz (1977) developed this particular version of Chamberlinian model of monopolistic competition, where each firm is assumed to be a negligible actor in that it has no impact on the overall market conditions. This is a behavioral assumption that firms neglect the piece of information on the impact of their outputs on the industry output. But why this behavioral assumption is a legitimate one, is never discussed clearly. Dixit and Stiglitz (1977) justifies that for a 'large number' of firms the individual firm's impact on the industry total output is negligible. Though it might be true for a continuum of firms, but for a finite number of firms, the individual firms do have an impact on the industry total output. This justification is heavily criticised by d'Aspremont et al. (1996) and found to be mathematically inconsistent by Keen and Standish (2006).

Later developments are made on the monopolistic competition and a more rigorous framework is presented in the works of Ottaviano et al. (2002) and Melitz and Ottaviano (2008). In this paper, this problem of monopolistic competition is addressed in quite a reversed way, where it is shown that in case of two firms, the circumstances might be such that even if the information on the impact of the individual output on the aggregate output is available, it is in best interest of the firms to strategically neglect that piece of information, much alike the works of Cellini et al. (2020). In their paper, Cellini et al. (2020) coined the term " strategic inattention" for the firms who neglected the information on the impact of their output on the aggregate output.

In the model described in this paper, the divisionalization game is same as that of Yuan (1999). The firms produce horizontally differentiated products, demand is linear, setting up new divisions is cost-less and the total cost of firms is normalized to 0. But, instead of the Cournot competition game that the divisions played earlier, now we analyse the impact if they are engaged in a monopolistic competition. Two types of "strategic inattention" arrives in this model, (i) divisions are unaware of the impact of their individual output on the mother firm's output, and (ii) divisions are unaware of the impact of their individual output on the industry total output. Though firms are setting up independent and autonomous divisions, but whether they will play the quantity game à la Cournot or they will opt for the monopolistic competition is determined by the firms. Whatever the firms decide, all it's divisions follow the same. We find that although there still remains the incentive to set up rival divisions for the firms, even in the case of "strategic inattention" but inattention among divisions reduces that incentive for the firms than the normal Cournot competition. This impact is much similar to the impact of delegation contracts on divisionalization, as discussed in González-Maestre (2000).

This paper considers a market with two firms, who are profit maximizers and play a non-cooperative three-stage game. In the first stage the firms simultaneously decide whether or not to let their divisions use the information that (i) the total firm's output is the sum of the individual division's output, or (ii) the total industry output is the sum of the individual division's output. Whatever the firm decides, all of it's divisions follows the same. That is, each firm first decides whether to make it's divisions attentive or inattentive towards (i) it's aggregate total output, or (ii) industry total output. In the second stage, the firms decide on the number of divisions simultaneously. In the third and the final stage, the divisions play a quantity competition game. This paper shows that one firm deciding to be inattentive to the information available and the other firm using that information, is the equilibrium outcome. Thus inattentive and attentive firms coexist in a Subgame Perfect Nash Equilibrium. This result is in sharp contrast to the findings of Cellini et al. (2020).

Thus here, we see that even if the information on the impact of the individual division's output on the mother firm or on the total industry output is available, it's in the best interest for one of the two firms to let their divisions ignore this piece of information. In this regard, this paper contributes to the stream of literature on information economics where it is argued that information is not very relevant as long as it helps in improving the best response to the rivals Kamien et al. (1990), Bassan et al. (2003).

The rest of the paper is organized as follows. Section 2 presents the model and the results (a) in case each firm's divisions play the quantity competition à la Cournot and (b) when the divisions are strategically inattentive (any of the two types as mentioned earlier). In section 3, the market structure is discussed when two firms are deciding whether or not to let their divisions be attentive or inattentive towards the firm's total output and then decide on the number of the divisions. In section 4, the same market structure containing two firms is discussed but here the firms have all the three possible options to decide from, i.e. let the divisions be attentive, inattentive towards firm's output or inattentive towards industry total output. Section 5 concludes this paper.

2 The Model

Consider an industry with n-firms selling n-horizontally differentiated products. Each firm sets up δ_h number of divisions. We assume that the cost of setting up a new division is zero. Each of the divisions of the same firm face the same linear inverse demand function as follows, which is in line with Cellini et al. (2020):

$$p_{hj} = a - \beta q_h - \sigma Q, \qquad (h = 1, 2, ..., n; j = 1, 2, ..., \delta_h) \tag{1}$$

where, a,β and σ are positive parameters. p_{hj} is the price of the products produced by the j^{th} division of the h^{th} firm and q_h is total output of the h^{th} firm, from it's δ_h divisions, i.e. $q_h = \sum_{j=1}^{\delta_h} q_{hj}$. Whereas, $Q = \sum_{h=1}^{n} q_h$ is the total industry output. For the limiting cases $\beta = 0$ is the homogeneous products market and with $\sigma = 0$ implies that the firms are producing independent products.

We assume that the total cost of the firm is constant which is normalized to be equated to 0. Then the profit function of the i^{th} division of the h^{th} firm can be given as:

$$\pi_{hi} = (a - \beta q_h - \sigma Q)q_{hi} \tag{2}$$

We now define a two-stage game which the oligopolists play:

Stage I: Firms decide the number of divisions.

Stage II: Divisions play quantity competition game simulteneously and independently.

2.1 Second Stage of the game (Quantity Competiton)

We proceed to solve for the equilibrium strategy using backward induction method. We begin by solving the second-stage equilibrium for each of the cases:

2.1.1 Cournot-Nash Outcome:

If all divisions of all firms behave as Counot oligopolists, then they maximise their profit function:

$$\pi_{hi}^C = \left(a - \beta \sum_{j=1}^{\delta_h} q_{hj}^C - \sigma \sum_{h=1}^{n} \sum_{j=1}^{\delta_h} q_{hj}^C \right) q_{hi}^C \tag{3}$$

where, a superscipt C denotes, Cournot outcome. Using the standard procedure one can see that the individual division's output and profit is given by:

$$q^C = \frac{a}{\beta(\delta_h + 1) + \sigma(\delta + 1)} \quad \text{and} \quad \pi^C = \frac{a^2(\beta + \sigma)}{[\beta(\delta_h + 1) + \sigma(\delta + 1)]^2} \tag{4}$$

where, $\delta = \sum_{h=1}^{n} \delta_h$, is total number of divisions of all firms in the industry.

2.1.2 Inattentive towards Firm's output:

Next we consider the scenario when the divisions of each individual firms are unaware of their effect on the firm's aggregate output and hence on the price. We refer to this as "*Strategic Inattention towards*

Firm's output". The divisions maximise the following profit function:

$$\pi^F_{hi} = \left(a - \beta q_h - \sigma \sum_{h=1}^{n} \sum_{j=1}^{\delta_h} q^F_{hj} \right) q^F_{hi} \tag{5}$$

where a superscript F denotes inattentive towards firm's output. The divisions consider q_h to be a parameter and $\frac{\partial q_h}{\partial q_{hi}} = 0$. Upon following the standard procedures of maximisation, the first order condition is given by:

$$\frac{\partial \pi^F_{hi}}{\partial q_{hi}} = a - \beta q_h - \sigma(Q + q_{hi}) = 0 \tag{6}$$

which can be re-written as:

$$a - \beta \left(q_{hi} + \sum_{j \neq i}^{\delta_h} q_{hj} \right) - \sigma \left(q_{hi} + \sum_{k \neq h}^{n} \sum_{j \neq i}^{\delta_k} q_{kj} + q_{hi} \right) = 0 \tag{7}$$

and then solved imposing symmetry on outputs. The individual divisions quantity and profit is given by:

$$q^F = \frac{a}{\beta \delta_h + \sigma(\delta + 1)} \quad \text{and} \quad \pi^F = \frac{a^2 \sigma}{[\beta \delta_h + \sigma(\delta + 1)]^2} \tag{8}$$

where, $\delta = \sum_{h=1}^{n} \delta_h$, is total number of divisions of all firms in the industry.

2.1.3 Inattentive towards Industry Output:

Next we consider the scenario when the divisions of each individual firms are unaware of their effect on the industry's aggregate output and hence on the price. We refer to this as "*Strategic Inattention towards Industry output*". The divisions maximise the following profit function:

$$\pi^I_{hi} = \left(a - \beta \sum_{j=1}^{\delta_h} q^I_{hj} - \sigma Q \right) q^I_{hi} \tag{9}$$

where a superscript I denotes inattentive towards industry output. The divisions consider Q to be a parameter and $\frac{\partial Q}{\partial q_{hi}} = 0$. Upon following the standard procedures of maximisation, the first order condition is given by:

$$\frac{\partial \pi^I_{hi}}{\partial q_{hi}} = a - \beta(q_h + q_{hi}) - \sigma Q = 0 \tag{10}$$

which can be re-written as:

$$a - \beta \left(q_{hi} + \sum_{j \neq i}^{\delta_h} q_{hj} + q_{hi} \right) - \sigma \left(q_{hi} + \sum_{k \neq h}^{n} \sum_{j \neq i}^{\delta_k} q_{kj} \right) = 0 \tag{11}$$

and then solved imposing symmetry on outputs. The individual divisions quantity and profit is given by:

$$q^I = \frac{a}{\beta(\delta_h + 1) + \sigma \delta} \quad \text{and} \quad \pi^I = \frac{a^2 \beta}{[\beta(\delta_h + 1) + \sigma \delta]^2} \tag{12}$$

where, $\delta = \sum_{h=1}^{n} \delta_h$, is total number of divisions of all firms in the industry.

Lemma 1. *Given the no. of divisions δ_h for each firm and the total number of divisions from all the firms δ, it can be inferred $q^F \gtreqless q^I$ with $\beta \gtreqless \sigma$ and $q^F, q^I > q^C$. Further, if each of the firms sets up at least one division then, $\pi^I \gtreqless \pi^F$ with $\beta \gtreqless \sigma$ and $\pi^C \geq \pi^F, \pi^I$.*

If we consider $\eta = \frac{\sigma}{\beta}$, then η can be thought of as a pure demand parameter that inversely measures the extent of product differentiation. From Lemma 1, we see that if $\eta \leq 1$ then $q^F \geq q^I$ but $\pi^F \leq \pi^I$ and if $\eta \geq 1$ then $q^F \leq q^I$ but $\pi^F \geq \pi^I$. That means, in a highly differentiated product market ($\eta \leq 1$) inattentive towards the firm's output makes the divisions more aggressive in the quantity market, but the profit earned by them is less than being inattentive towards the industry output. Strategic Inattention, whether inattentive towards firm's output or industry output, makes the divisions more aggressive than being attentive towards both the outputs (normal Cournot-Nash equilibrium) but the divisions earn more profit if all them are attentive towards both the outputs rather than all them being strategically inattentive.

2.2 First Stage of the game (Divisionalization)

Given the characterization of the equilibrium in the second stage, we now solve for the divisionalization decision of the firms in the first stage:

2.2.1 Cournot Nash Outcome:

Under Cournot Nash equilibrium outcome, the profit of a single parent-firm is given by:

$$\pi_h^C = \frac{a^2(\beta+\sigma)\delta_h}{[\beta(\delta_h+1)+\sigma(\delta+1)]^2} \qquad (h = 1, 2, ..., n) \tag{13}$$

Firms maximise π_h^C with respect to δ_h to get the optimal number of divisions. Following the usual procedures we get the first order condition as:

$$\frac{\partial \pi_h^C}{\partial \delta_h} = 0 \implies \beta(\delta_h+1)+\sigma(\delta+1) = 2\delta_h(\beta+\sigma) \tag{14}$$

and then solved imposing symmetry in the number of divisions of each firms. The number of divisions set up by each firm, under Cournot Nash equilibrium outcome is:

$$\delta^C = \frac{\beta+\sigma}{\beta+2\sigma-n\sigma} \tag{15}$$

2.2.2 Inattentive towards Firm's output:

When the divisions are inattentive towards the parent-firm's output, the total profit of a single parent-firm is given by:

$$\pi^F = \frac{a^2\sigma\delta_h}{[\beta\delta_h+\sigma(\delta+1)]^2} \qquad (h = 1, 2, ..., n) \tag{16}$$

Similarly, as the case for Cournot Nash Outcome, the firms maximise π_h^F with respect to δ_h to arrive at the following first order condition:

$$\frac{\partial \pi_h^F}{\partial \delta_h} = 0 \implies \beta\delta_h+\sigma(\delta+1) = 2\delta_h(\beta+\sigma) \tag{17}$$

Upon solving for the equilibrium by imposing symmetry, we get the number of divisions set up by each of the firms when the all the divisions are inattentive towards firm's output, to be:

$$\delta^F = \frac{\sigma}{\beta+2\sigma-n\sigma} \tag{18}$$

2.2.3 Inattentive towards Industry Output:

When the divisions are inattentive towards the total industry output but takes into consideration the effect of it's output on the parent-firm's total output, the profit of a single parent-firm is given by:

$$\pi_h^I = \frac{a^2\beta\delta_h}{[\beta(\delta_h+1)+\sigma\delta]^2} \qquad (h=1,2,......,n) \tag{19}$$

Following the usual procedures as above the first order condition can be written as:

$$\frac{\partial\pi_h^I}{\partial\delta_h} = 0 \implies \beta(\delta_h+1)+\sigma\delta = 2\delta_h(\beta+\sigma) \tag{20}$$

Solving for the equilibrium number of divisions, we get the number of divisions set up by each firm when all the divisions are inattentive towards the total industry output as:

$$\delta^I = \frac{\beta}{\beta+2\sigma-n\sigma} \tag{21}$$

Summing up both the stages, the output (of each division) & division pair for all the three cases can be written as:

$$(q^C,\delta^C) = \left(\frac{a(\beta+2\sigma-n\sigma)}{2(\beta+\sigma)^2}, \frac{\beta+\sigma}{\beta+2\sigma-n\sigma}\right) \tag{22}$$

$$(q^F,\delta^F) = \left(\frac{a(\beta+2\sigma-n\sigma)}{2\sigma(\beta+\sigma)}, \frac{\sigma}{\beta+2\sigma-n\sigma}\right) \tag{23}$$

$$(q^I,\delta^I) = \left(\frac{a(\beta+2\sigma-n\sigma)}{2\beta(\beta+\sigma)}, \frac{\beta}{\beta+2\sigma-n\sigma}\right) \tag{24}$$

And the profit earned by the firms is given by

$$\pi^C = \pi^F = \pi^I = \frac{a^2(\beta+2\sigma-n\sigma)}{4(\beta+\sigma)^2} \tag{25}$$

Lemma 2. *In the given setup $q^F \gtreqless q^I$ with $\beta \gtreqless \sigma$ and $q^F, q^I > q^C$. In case of the equilibrium number of divisions set up by the firms, $\delta^I \gtreqless \delta^F$ with $\beta \gtreqless \sigma$ and $\delta^C > \delta^F, \delta^I$. Finally, $\pi^C = \pi^F = \pi^I$.*

From Lemma 2 we can infer that as seen in Lemma 1, even if the number of divisions is decided strategically by the firms, in a highly differentiated product market, i.e. with $\eta \leq 1$ inattentive towards firm's output make the divisions more aggressive in the quantity market than being inattentive towards the total industry output and vice versa. Also, any form of inattention is making the firms more aggressive which is in line with the results of Cellini et al. (2020).

But we see that, with $\eta \leq 1$, the product market being highly differentiated the incentive to divisionalize is less when the divisions are inattentive towards firm's output than when they are inattentive towards the industry output and vice versa. Although, any form of inattention as a whole, takes away the incentive to divisionalize, which is similar to the effect of delegation contract on divisionalization as shown in González-Maestre (2000). Hence, there lies an intrinsic relation between delegation contracts and inattention, even in case of divisionalization. Looking at the final profit earned by the firms, we see that they are same. Since strategic interaction between firms takes place here in a two-fold way, first in the form of strategic divisionalization and then in the form of quantity competition, the firms adjust their output and divisions in a manner that in equilibrium they are earning the same profit in all of the three cases.

3 Market Structure: Inattention (F) Vs Attention (C)

Next we consider the same differentiated product market with 2-firms. Now we consider the following three-stage game:

Stage I: Firms decide whether all of it's divisions will be inattentive or attentive towards firm's output, while playing the quantity game in Stage-III.

Stage II: Firms decide the number of divisions strategically.

Stage III: Divisions play quantity competition game simultaneously and independently.

We search for the subgame perfect equilibria focusing on the equilibrium outcomes.

Proposition 1. *One firm deciding, it's divisions to be inattentive towards the firm's output and the other firm deciding, it's divisions to be attentive is an unique Subgame Perfect Nash Equilibrium in a duopoly.*

Proof. Let Firm-1 decides to make all of it's divisions inattentive to the Firm's output and Firm-2 decides to remain attentive. We start solving by the usual backward induction method.

Then, in Stage-III the necessary conditions that characterizes the equilibrium are given accordingly as,

$$a - \beta q_1 - \sigma(Q + q^F) = 0 \tag{26}$$

$$a - \beta(q_2 + q^C) - \sigma(Q + q^C) = 0 \tag{27}$$

where, q_1, q_2 are the total outputs of Firm-1 and Firm-2, respectively and q^F, q^C represents the equilibrium output choice of strategically inattentive and attentive divisions, respectively. Then, $q_1 = \delta_1 q^F$ and $q_2 = \delta_2 q^C$ with δ_1 representing the number of divisions of Firm-1 and δ_2 represents that of Firm-2. The total industry output is given by $Q = \delta_1 q^F + \delta_2 q^C$.

Upon solving (26) and (27), we get the following equilibrium outputs:

$$q^F = \frac{a\left(\beta\delta_2 + \beta + \sigma\right)}{\gamma} \quad \text{and} \quad q^C = \frac{a\left(\beta\delta_1 + \sigma\right)}{\gamma} \tag{28}$$

where, $\gamma = \sigma\left(\delta_2 + 1\right)\left(\beta + \sigma\right) + \delta_1\left[\beta(\beta + 2\sigma)\delta_2 + (\beta + \sigma)^2\right]$

In the Stage-II (the divisionalization stage), to decide the number of divisions the two firms maximise their profits given by,

$$\pi_1^F = \frac{a^2\delta_1\sigma\left(\beta\delta_2 + \beta + \sigma\right)^2}{\gamma^2} \quad \text{and} \quad \pi_2^C = \frac{a^2\delta_2(\beta + \sigma)\left(\beta\delta_1 + \sigma\right)^2}{\gamma^2} \tag{29}$$

Where, the superscripts F and C are for Firm-1 asking it's divisions to be inattentive and for Firm-2 asking it's divisions to be attentive, respectively.

Upon following the usual procedures of maximisation, we arrive at the following divisionalization decisions,

$$\delta_1 = \frac{\sigma}{\sqrt{\beta(\beta + 2\sigma)}} \quad \text{and} \quad \delta_2 = \frac{\beta + \sigma}{\sqrt{\beta(\beta + 2\sigma)}} \tag{30}$$

Then using the equilibrium number of divisions as obtained in (30), the equilibrium profits of both the firms can be given by:

$$\pi_1^F = \frac{a^2\sqrt{\beta(\beta + 2\sigma)}}{\left(\sqrt{\beta(\beta + 2\sigma)} + \beta + 2\sigma\right)^2} = \pi_2^C \tag{31}$$

From our analysis in the previous section, if we put n=2, we get that

$$\pi^F = \pi^C = \frac{a^2\beta}{4(\beta+\sigma)^2} \tag{32}$$

Then in Stage-I, the game that the firms face can be necessarily be represented by the following 2×2 payoff matrix given in Figure 1.

		Firm 2	
		F	C
Firm 1	F	(π^F, π^F)	(π_1^F, π_2^C)
	C	(π_2^C, π_1^F)	(π^C, π^C)

Figure 1: Payoff Matrix for Inattention Vs Attention

The solution of the game as represented by the payoff matrix is (π_1^F, π_2^C) and (π_2^C, π_1^F), since for all $\beta > 0, \sigma > 0, a > 0$ it can be verified that $\pi_2^C = \pi_1^F > \pi^F = \pi^C$.

Hence, one firm choosing to be attentive and the other choosing to be inattentive is an unique Subgame Perfect Nash Equilibrium.

□

Thus while the market regime is determined endogenously by the strategic choices of the two firms, one of the firms decides to use the information on the impact of the individual output of their divisions on the their (read, the firm's) total output whereas the other firm neglects that information. This is the unique SPNE. Clearly, the firms have the incentive to set up rival divisions which leads to an interior SPNE. The number of divisions depends on the extend of differentiation determined by η. Setting up divisions in the market, both the firms increase the degree of competition, thus sustaining the coexistence of attentive divisions and inattentive divisions in the market.

In their analysis, Cellini et al. (2020) stated that there exists bounds below which all firms are attentive in equilibrium and above which all firms are inattentive in equilibrium, and the outcome where some firms behave as oligopolists and others as monopolistic competitors never arise as a SPNE in pure strategies. But, here we see that under strategic divisionalization for two firms, both of them being inattentive or attentive is never a subgame perfect equilibrium, rather one of them is behaving as an oligopolist and the other as a monopolistic competitor. Interestingly, this equilibrium is sustained under as many as divisions (which are strategically being determined by the firms) operating in the market, without any restriction being imposed upon them.

4 Market Structure: Inattention (F) Vs Inattention (I) Vs Attention (C)

The timing of events remain the same as in previous section. We extend our analysis to the case when the firms have the options to choose from

(i) inattentive to firm's output (F)

(ii) inattentive to total industry output (I)

(iii) attentive to both the outputs (C)

Proposition 2. *One firm deciding it's divisions to be inattentive towards the firm's output (or, total industry output) and the other firm deciding it's divisions to be attentive; or One firm deciding it's divisions to be inattentive towards the firm's output (industry total output) and the other firms deciding it's divisions to be inattentive towards industry total output (firm's output) are Subgame Perfect Nash Equilibria.*

Proof. We already know the profits of the firms if one of them chooses to be inattentive to firm's output (F) and the other remains attentive to both the outputs (C). So, next we consider the case when one of them chooses to be inattentive to industry total output and the other attentive towards both the outputs.

Suppose Firm-1 decides to make all of it's divisions inattentive towards industry total output (I) and Firm-2 decides to remain attentive (C). We solve by the backward induction method.

Accordingly in Stage-III the necessary conditions that characterizes the equilibrium are,

$$a - \beta(q_1 + q^I) - \sigma Q = 0 \tag{33}$$

$$a - \beta(q_2 + q^C) - \sigma(Q + q^C) = 0 \tag{34}$$

where, q_1, q_2 are as before, total outputs of Firm-1 and Firm-2, respectively and q^I, q^C represents the equilibrium output choice of strategically inattentive and attentive divisions, respectively. Then, $q_1 = \delta_1 q^I$ and $q_2 = \delta_2 q^C$ with δ_1 representing the number of divisions of Firm-1 and δ_2 represents that of Firm-2. The total industry output is given by $Q = \delta_1 q^I + \delta_2 q^C$.

Upon solving (33) and (34), we get the following equilibrium outputs:

$$q^I = \frac{a\left(\beta\delta_2 + \beta + \sigma\right)}{\tau} \text{and} \quad q^C = \frac{a\beta\left(\delta_1 + 1\right)}{\tau} \tag{35}$$

where, $\tau = \beta\left(\delta_2 + 1\right)\left(\beta + \sigma\right) + \delta_1\left[\beta(\beta + 2\sigma)\delta_2 + (\beta + \sigma)^2\right]$

In Stage-II, to decide the number of divisions the two firms maximise their profits given by,

$$\pi_1^I = \frac{a^2\beta\delta_1\left(\beta\delta_2 + \beta + \sigma\right)^2}{\tau^2} \quad \text{and} \quad \pi_2^C = \frac{a^2\beta^2\delta_2(\beta + \sigma)\left(\delta_1 + 1\right)^2}{\tau^2} \tag{36}$$

Where, the superscripts I and C are for Firm-1 asking it's divisions to be inattentive and for Firm-2 asking it's divisions to be attentive, respectively.

Upon following the usual procedures of maximisation, we arrive at the following divisionalization decisions,

$$\delta_1 = \frac{\beta}{\sqrt{\beta(\beta + 2\sigma)}} \quad \text{and} \quad \delta_2 = \frac{\beta + \sigma}{\sqrt{\beta(\beta + 2\sigma)}} \tag{37}$$

Then using the equilibrium number of divisions as obtained in (37), the equilibrium profits of both the firms can be given by:

$$\pi_1^I = \frac{a^2\sqrt{\beta(\beta + 2\sigma)}}{\left(\sqrt{\beta(\beta + 2\sigma)} + \beta + 2\sigma\right)^2} = \pi_2^C \tag{38}$$

which is similar to the expressions as in (31).

It can be verified that when one firm chooses to be inattentive towards firm's output and the other towards the industry output, the profit is given by the expressions (38).

Let us denote the profit expressions from (38) and (32) as:

$$\tilde{\pi} = \frac{a^2\sqrt{\beta(\beta+2\sigma)}}{\left(\sqrt{\beta(\beta+2\sigma)}+\beta+2\sigma\right)^2} \quad \text{and} \quad \bar{\pi} = \frac{a^2\beta}{4(\beta+\sigma)^2} \tag{39}$$

Then in Stage-I, the game that the firms face is represented by the following 3×3 payoff matrix given in Figure 2.

		Firm 2		
		F	C	I
Firm 1	F	$(\bar{\pi}, \bar{\pi})$	$(\tilde{\pi}, \tilde{\pi})$	$(\tilde{\pi}, \tilde{\pi})$
	C	$(\tilde{\pi}, \tilde{\pi})$	$(\bar{\pi}, \bar{\pi})$	$(\tilde{\pi}, \tilde{\pi})$
	I	$(\tilde{\pi}, \tilde{\pi})$	$(\tilde{\pi}, \tilde{\pi})$	$(\bar{\pi}, \bar{\pi})$

Figure 2: Payoff Matrix for Inattention Vs Inattention Vs Attention

As in the previous section, for all $\beta > 0, \sigma > 0, a > 0$ it can be verified that $\tilde{\pi} > \bar{\pi}$. Hence, $\{(h,k) : h,k = F,C,I; h \neq k\}$ is the set of all pure strategy Subgame Perfect Nash Equilibria of the game. □

It was seen in the previous section that in equilibrium one of the firms was using the information of the impact of the division's output on the firm's total output and the other was deliberately ignoring it strategically. Here too, in equilibrium coexistence of attentive and inattentive firms is established. But along with that, since now the firms can also choose from the two types of inattention, in equilibrium both the firms can be inattentive with different types of inattention. Thus, with divisionalization the competition increases and both the firms being attentive is never sustained in equilibrium for a duopoly.

Corollary 1. *Even if firms were choosing between inattentive towards industry total output or being attentive, then also one being inattentive towards the industry total output and the other being attentive would have been the Subgame Perfect Nash Equilibrium.*

Hence, irrespective of the type of inattention that the firm wants to choose from, the endogenous market structure determined by the strategic interaction of the two firms, corresponds to a coexistence of an inattentive and an attentive firm. This deviation from the result of Cellini et al. (2020) is credited to divisionalization. Firms can now divisionalize and strategically decide on the number of divisions, which gives rise to higher competition. In this framework, it is sustainable for one of the firms to setup inattentive divisions and the other firm to setup attentive divisions.

5 Conclusion

In this paper a differentiated product market is modelled, where firms set up independent competitive divisions who plays a quantity competition game among themselves. The firms also decide whether or not, the divisions should use the information regarding the impact of their individual output on the firm's output or the total industry output. When divisions are using the information, its the regular oligopolistic competition à la Cournot that everyone is aware of, but when they are intentionally ignoring that information of their impact on the firm's output or on the total industry output, it's like the monopolistic competition. The firms have the incentive to divisionalize and set up independent competitive divisions, even during monopolistic competition. Thus, no matter what sort of competition takes place in the market, there's always incentive for the firms to divisionalize. Although, the incentive to divisionalize decreases for the case of monopolistic competition, similar to the effect of delegation contract on divisionalization incentives.

For a duopoly, a three stage game is considered, where in the first stage the firms decide simultaneously and independently whether all of it's respective divisions will use the information regarding the division's impact on the output of the firm or on the industry. In the next two stages the divisionalization and the quantity competition takes place, respectively. It's seen that for any value of the parameter one firm deciding to use the information and the other ignoring it, is the Subgame Perfect Nash Equilibrium in pure strategy. Thus inattentive and attentive firms coexists in the equilibrium, for a duopoly in the case of duopoly. If the analysis is extended to the case where the firms can now choose from three options: two types of inattention and being attentive, it's seen that in the subgame perfect Nash equilibrium either both of them will be inattentive but of opposite types or one of them will be inattentive and the other attentive. Hence in equilibrium, one can never get a duopoly market with divisionalization such that both the firms will ask it's divisions to be attentive towards firm's output or industry output.

The analysis done in this paper is with respect to a market of two firms. The motive of this paper has been to analyse the coexistence of inattentive and attentive firms in the equilibrium, which is sufficed by a duopoly. One can extend this analysis to a more general case with finite number of firms and find out exact the equilibrium outcomes. Secondly, while deriving the divisionalization outcomes it is assumed that the division is a continuous variable, which is a very general assumption with respect to the literature. But, it would be interesting to continue with the divisionalization derivations, by considering that the divisions is discrete and analyse the findings henceforth.

References

Bassan, B., O. Gossner, M. Scarsini, and S. Zamir (2003). Positive value of information in games. *International Journal of Game Theory 32*(1), 17–31.

Baye, M., K. Crocker, and J. Ju (1996, March). Divisionalization, franchising, and divestiture incentives in oligopoly. *American Economic Review 86*(1), 223–236.

Cellini, R., L. Lambertini, and G. I. Ottaviano (2020). Strategic inattention, delegation and endogenous market structure. *European Economic Review 121*, 103324.

Corchón, L. C. (1991). Oligopolistic competition among groups. *Economics Letters 36*(1), 1–3.

d'Aspremont, C., R. D. S. Ferreira, and L.-A. Gérard-Varet (1996). On the dixit-stiglitz model of monopolistic competition. *The American Economic Review*, 623–629.

Dixit, A. K. and J. E. Stiglitz (1977). Monopolistic competition and optimum product diversity. *The American economic review 67*(3), 297–308.

González-Maestre, M. (2000). Divisionalization and delegation in oligopoly. *Journal of Economics & Management Strategy 9*(3), 321–338.

Kamien, M. I., Y. Tauman, and S. Zamir (1990). On the value of information in a strategic conflict. *Games and Economic Behavior 2*(2), 129–153.

Keen, S. and R. Standish (2006). Profit maximization, industry structure, and competition: A critique of neoclassical theory. *Physica A: Statistical Mechanics and its Applications 370*(1), 81–85.

Melitz, M. J. and G. I. P. Ottaviano (2008, 01). Market Size, Trade, and Productivity. *The Review of Economic Studies 75*(1), 295–316.

Ottaviano, G., T. Tabuchi, and J.-F. Thisse (2002). Agglomeration and trade revisited*. *International Economic Review 43*(2), 409–435.

Polasky, S. (1992). Divide and conquer on the profitability of forming independent rival divisions. *Economics Letters 40*(3), 365–371.

Sloan, A. P. (1990). *My years with general motors.* Currency.

Williamson, O. E. (1973). Markets and hierarchies: some elementary considerations. *The American economic review 63*(2), 316–325.

Yuan, L. (1999). Product differentiation, strategic divisionalization, and persistence of monopoly. *Journal of Economics & Management Strategy 8*(4), 581–602.