WP-2022-010

Minimum Inequality Taxation, Average and Minimally Progressive Taxations and Depolarization

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Indira Gandhi Institute of Development Research, Mumbai
August 2022

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JEL Code: D31, D63, H24

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Abstract

An inequality minimizing taxation (IMT) policy addresses the problem of procuring a certain amount of tax from a given set of persons in an inequality minimizing manner by maintaining rank orders of the individuals in the pre- and post-tax situations and without imposing any notion of inequality invariance. In this article we demonstrate analytically that the newly introduced IMT policy is sufficient but not necessary for average progressive and minimally progressive taxation principles. Using a recent result from the literature we then show that an IMT scheme also implies but is not implied by depolarizing (bipolarization reducing) and bipolarization minimizing taxation policies. An empirical illustration of our results is provided using income data collected by the Center for Monitoring Indian Economy.

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1. Introduction

Relationship between income inequality and progressive income taxation, non-decreasingness of average tax liability with income (Musgrave and Thin ,1948), has been investigated rigorously by many authors, including Fellman (1976), Jakobsson (1976), Kakwani (1977), Eichhorn et al. (1984) and Le Breton et al. (1996). Following Moyes (1988) we refer to this notion progressive tax principle as 'average progressive taxation' (APT). The underlying notion of inequality in this context is of relative type; an equi-proportionate change in all incomes leaves inequality unchanged. While relative inequality invariance represents a particular notion of value judgment, an alternative to this is absolute inequality invariance, which requires inequality to remain unaltered under equal absolute changes in all incomes (Kolm, 1976). Moyes (1988) established the link between absolute income inequality and 'minimally progressive taxation' (MPT), non-decreasingness of tax liability with income. The results of Eichhorn et al. (1984) and Moyes (1988) also rely on the assumption that taxes are incentive preserving; a person with a higher pre-tax income than another cannot have a lower post-tax income as well. Thus, the incentives for the individuals to earn more are maintained under this assumption (Fei, 1981).

Both APT and MPT are quite appealing as taxation rules in that sense that each of them does not increase inequality in its respective sense. While APT does not raise inequality by cutting relative income differentials, MPT does so by slashing absolute income differentials. Thus, the two notions of progressivity are explicitly sensitive to specific concepts of inequality invariance.

It may often be worthwhile to investigate how the middle income group of a society gets affected by a progressive taxation since '…the best political economy is formed by citizens of the middle class' Aristotle (-350). A large and rich middle class of a society makes a significant contribution to the society's tax revenue. More generally, a rich and large middle class of a society contributes to the well-being of the society in many ways, including higher economic growth, better infrastructure, provisions of new public goods, and becoming a key provider of highly educated/trained professionals (e.g., doctors, technologists and skilled labor)[1]. Since 1900s social scientists have attempted to relate the notion of bipolarization, the 'shrinking middle

[1] See also, among others, Birdsall (2007), McBride et al. (2011), Chakravarty (2015) and Duclos and Tapture (2015).

class', to the size of the middle class[2]. An income distribution which is more spread out from the middle, so that there are fewer persons in the middle income group than in the two extreme positions, is known as more bipolarized (Wolfson, 1994). This is also referred to as 'two-peaks' or 'two-components' hypothesis (Quah , 1996).Thus, while inequality deals with the dispersion of incomes among all the individuals, bipolarization is concerned with the income distributions of two polar modes on both sides of the median.

In a highly interesting article Carbonell-Nicolau and Llavador (2021) provided an alternative normative justification for a taxation principle; progressivity relying on equity and depolarization.They showed that, although inequality and bipolarization are intrinsically two different postulations, taxes are progressive in the sense of non-increasingness of relative inequality if and only they are not (relative) bipolarization augmenting.

As stated above, although the principles APT and MPT do not heighten inequality; they not be inequality minimizing. Often a policymaker's objective may be to raise the same amount of tax, as raised under the scheme average progressive taxation or minimally progressive taxation, by seeking inequality reduction to the maximum possible extent. In a recent contribution Chakravarty and Sarkar (2022) addressed the problem of collecting a certain amount of tax in an inequality minimizing manner by maintaining the incentive preservation assumption. They demonstrated that this moral purpose can be realized by employing all inequality metrics that fulfill two essential postulates. No notion of inequality invariance is required for this general result to hold. In fact, all inequality yardsticks that register non-decreasingness of inequality under APT and MPT verify these two basic postulates.

Given that each of the above taxation policies is highly appealing from an egalitarian perspective, it will certainly be worthy to investigate interrelationships among them. This is precisely the objective of the present article. We demonstrate rigorously that the three taxation principles, namely, APT, MPT and depolarizing taxation criteria are implied by the inequality minimizing taxation (IMT) policy. But none of the principles implies the IMT schedule.

Assuming that social welfare is expressed as a trade-off between equity and efficiency, we also look at the sizes of welfare gain when we adopt IMT over APT and MPT. The specific inequality indices we choose for this purpose satisfy a compromise property, when multiplied by

[2] For discussions on the size of middle class, see Thurow (1984), Davies and Huston (1992), Easterly (2001), Duclos and Tapture (2015) and others.

the mean income, the relative indices become indices of absolute category. This property of the underlying inequality metric enables us to use the same welfare function that can be related to the selected relative and absolute inequality indices in a monotonically decreasing manner. For instance, the well-known Gini index of inequality possesses this compromise property and the same Gini welfare function can be related to the relative and absolute forms of the Gini in a negative monotonic way. Consequently, comparison of welfare gains across taxation principles become meaningful.

After discussing the preliminaries in the next section, Section 3 presents the background and motivations for the problem addressed in the paper. The subject of Section 4 is a rigorous analysis on the relationships among different notions of taxation considered in the paper. Then in Section 5 we provide an empirical illustration of our results using income data collected by the *Centre for Monitoring Indian Economy.* Finally, Section 6 concludes.

2. Preliminaries

An income distribution in a society consisting of n individuals is represented by vector $x=(x_1, x_2, ...,x_n)$, where $x_i > 0$ is the income of person $i, i=1,2,...,n$. An income distribution $x=(x_1, x_2, ...,x_n)$ is said to non-decreasingly ordered if $x_1 \leq x_2 \leq ..... \leq x_n$. We write D^n_{++} for the set of all non-decreasingly ordered income distributions in an n person society, where all incomes are positive.

We assume at the outset inequality in $(x_1, x_2, ...,x_n)$ is same as the inequality in $\left(x_{\pi(1)}, x_{\pi(2)}, ...,x_{\pi(n)}\right)$, where π is a reordering of $x=(1,2,.......,n)$. Equivalently, we say that the underlying inequality index satisfies anonymity. This property ensures that any characteristic other than income is irrelevant to the evaluation of inequality. It is, therefore, sufficient to define an inequality metric on D^n_{++}. An inequality index I is a non-constant, non-negative real valued function defined on the set of all income distributions. Formally, $I \bigcup_{n \geq 2} D^n_{++} \rightarrow R_+$, where R_+ is the non-negative part of the real line R. In the remainder of our analysis, unless specified, we will deal with a fixed population size $n > 1$. For any $x \in D_{++}^{n}$, we write $\mu(x)$ for the mean $\frac{1}{n}\sum_{i=1}^{n} x_i$ of x.

For any $y \in D_{++}^{n}$, $x \in D_{++}^{n}$ is said obtained from y by a progressive transfer if for some pair (i, j), with $y_i < y_j$, $x_i = y_i + c \leq x_j = y_j - c$, $c > 0$ and $x_k = y_k$ for all $k \neq i, j$. In words, the distribution x is obtained from the distribution y by a transfer of a positive amount of income c from person j to the poorer person i such that the transfer does not make i richer than j and all other incomes remain unchanged. An inequality metric is said to fulfill the Pigou-Dalton transfer principle (transfer principle, for short) if $I(x) \leq I(y)$. Anonymity and the transfer principle are treated as minimal postulates for an inequality quantifier. Under anonymity only rank preserving transfers can take place. An inequality index satisfying anonymity and the transfer principle is S-convex (Dasgupta et al. 1973)[3]. In addition to the two basic postulates we also assume a normalization condition which stipulates that for $x \in D_{++}^{n}$, $I(x) = 0$ if and only if x is perfectly equal.

An inequality standard is relative or absolute according as it is scale or translation invariant. Formally $I : D_{++}^{n} \to R_{+}$ is relative if for all $x \in D_{++}^{n}$, $I(cx) = I(x)$, where $c > 0$ is any scalar and I is absolute if $I(x + c1^{n}) = I(x)$, where c is a scalar such that $x + c1^{n} \in D_{++}^{n}$ and 1^{n} is the *n*-coordinated vector of ones.

To investigate the welfare implications of our analysis in a later section, we will need to choose specific relative inequality metrics with the compromise property. The Gini index I_G, defined in terms of absolute values of pairwise income differences, is one such index:

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[3] Analytically, a function $f : D_{++}^{n} \to R$ is called S-convex if for all $x \in D_{++}^{n}$ and for all bistochastic matrices Q of order n, $f(xQ) \leq f(x)$, where a bistochastic matrix of order n is an $n \times n$ matrix with non-negative entriess , with each of its rows and columns sums being equal to one. $f : D_{++}^{n} \to R$ is S-concave if $-f$ is S-convex(see Marshall et al. , 2011).

$$I_G(x) = \frac{1}{2n^2 \mu(x)} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| x_i - x_j \right| , \tag{1}$$

where $x \in D_{++}^n$ is arbitrary. Since incomes are non-decreasingly ordered, we can rewrite $I_G(x)$ as

$$I_G(x) = 1 - \frac{1}{n^2 \mu(x)} \sum_{i=1}^{n} [2(n-i)+1] x_i . \tag{2}$$

An attractive feature of I_G is that it can be expressed as twice the area enclosed between the Lorenz curve and the line of equality. The ordinate of the Lorenz curve of a given distribution $x \in D_{++}^n$ at the population proportion $\frac{j}{n}, 1 \leq j \leq n$, is given by $L\left(x, \frac{j}{n}\right) = \frac{\sum_{i=1}^{j} x_i}{n\mu(x)}$. Then the Lorenz curve $L(x, p)$ of x is defined by setting $L(x, 0) = 0$ and

$$L\left(x, \frac{j+\alpha}{n}\right) = (1-\alpha) L\left(x, \frac{j}{n}\right) + \alpha L\left(x, \frac{j+1}{n}\right) \tag{3}$$

for all $\alpha \in [0,1]$.

The second inequality metric we will consider is the recently revived Bonferroni index I_B which, for any $x \in D_{++}^n$, is defined in terms of its partial means $\frac{1}{i} \sum_{j=1}^{i} x_j , i = 1, 2, \ldots\ldots, n$.

Formally,

$$I_B(x) = 1 - \frac{1}{n\mu(x)} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{i} x_j . \tag{4}$$

Graphically, $I_B(x)$ is the area between the Bonferroni curve of x and the horizontal line at 1. The ordinate of the Bonferroni curve of the distribution $x \in D_{++}^n$ at the population proportion

$\frac{j}{n}, 1 \leq j \leq n,$ is given by $B\left(x, \frac{j}{n}\right) = \frac{\frac{\sum_{i=1}^{j} x_i}{n\mu(x)}}{\frac{j}{n}}$. The curve $B(x, p)$ is then defined by letting $B(x,0) = 0$ and

$$B\left(x, \frac{j+\alpha}{n}\right) = BL\left(x, \frac{j}{n}\right) + (1-\alpha)B\left(x, \frac{j+1}{n}\right) \tag{5}$$

for all $p \in [0,1]$. (See Aaberge , 2007 , Bárcena-Martin and Silber, 2013 and Chakravarty and Sarkar, 2021.)

With a given rank order of incomes both I_G and I_B are linear. This property of the two metrics enables us to convert them into their absolute counterparts A_G and A_B by multiplying with the mean income. Formally, the absolute variants of the Gini and Bonferroni indices are defined respectively as:

$$A_G(x) = \mu(x) - \frac{1}{n^2} \sum_{i=1}^{n} [2(n-i)+1]x_i \ , \tag{6}$$

and

$$A_B(x) = \mu(x) - \frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{i} x_j \ . \tag{7}$$

The social welfare functions associated with I_G is defined using the well-known Atkinson (1970)-Kolm (1969)-Sen (1973) form:

$$\begin{aligned} W_G(x) &= \mu(x)\left(1 - I_G(x)\right) \\ &= \frac{1}{n^2} \sum_{i=1}^{n} ((2n-i)+1)x_i \ , \\ &= \mu(x) - A_G(x) \end{aligned} \tag{8}$$

where $x \in D^n_{++}$ is arbitrary (see also Blackorby and Donaldson, 1978, Donaldson and Weymark, 1980 and Weymark, 1981). Likewise, for I_B the welfare function is defined as

$$\begin{aligned} W_B(x) &= \mu(x)(1 - I_B(x)) \\ &= \frac{1}{n} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{i} x_j \\ &= \mu(x) - A_B(x) \end{aligned} \quad , (9)$$

where $x \in D^n_{++}$ is arbitrary (. Chakravarty and Sarkar (2021) provided a systematic comparison between W_G and W_B.).

Thus, W_G (respectively W_B) has a decreasing relationship with both I_G and A_G (respectively I_B and A_B). The Gini and the Bonferroni welfare functions, W_G and W_B, are S-concave and non-decreasing in individual incomes. They are linear homogenous and unit translatable, where unit translatability of a welfare function demands that if the same amount of income is added to all the incomes then welfare increases by the amount itself[4].

In equations (8) and (9) social evaluation of income distributions have been represented in terms of trade-off between efficiency (mean income) and equity. When efficiency considerations are absent (mean income is fixed), an increase in W_G (respectively W_B) is equivalent to a reduction in I_G and A_G (respectively I_B and A_B) and vice-versa.

We conclude this section with a brief analytical discussion on bipolarization. This notion of polarization is concerned with the spread and dispersion of the distribution of income from the middle position, the median. For any $x \in D^n_{++}$, we denote the median of x by $m(x)$. If n is odd, $m(x)$ is the $\frac{(n+1)^{th}}{2}$ observation in x . But if n is even, the arithmetic mean of the $\frac{n^{th}}{2}$

[4] Social welfare functions possessing these two characteristics are called distributionally homogenous. Such welfare functions become helpful in measuring the economic distance between two income distributions that reflects the well-being of one population relative to that of another (Chakravarty and Dutta, 1987). Bossert (1990) used distributional homogeneity as an axiom to characterize the 'single-series Ginis'.

and the $\frac{(n+1)^{th}}{2}$ observations in x is taken as the median. For instance, if $x=(1,2,3,4,5)$ and $y=(1,2,3,4,5,6)$, then $m(x)=3$ and $m(y)=3.5$. For any $x \in D^n_{++}$ let x_- and x_+ stand respectively for the subvectors of x that include x_i for $x_i \le m(x)$ and $x_i > m(x)$, respectively. Thus, any $x \in D^n_{++}$ can be written in terms of the two subvectors as $x=(x_-, x_+)$ if n is even and $x=(x_-, m(x), x_+)$ if n is odd. For any $x, y \in D^n_{++}$, by $x \ge y$ we mean that $x_i \ge y_i$ for all $i=1,2,......,n$, with > for at least one i. Also for any $x, y \in D^n_{++}$, we use the abbreviation xEy to indicate that x has been deduced from y by a progressive transfer.

For a population of size n a bipolarization index P is a non-constant, non-negative real valued function defined on its income space D^n_{++}. Formally, $P: D^n_{++} \to R_+$ .Two characteristics that are treated as being innate to the notion of bipolarization are increased spread and increased bipolarity [5] . While the former demands that a reduction (respectively an increment) in any income below (respectively above) the median does not decrease bipolarization, the latter claims that an egalitarian transfer on the either side of the median does not decrease bipolarization. Thus, these features are concerned respectively with movements of the individuals away from the median and clustering of persons on the same side of the median. They represent respectively 'alienation' and 'identification' components of bipolarization[6]. The characteristic increased bipolarity shows that inequality and bipolarization are two different concepts.

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[5] See, among others, Wolfson (1994), Wang and Tsui (2000), Bossert and Schworm (2008), Chakravarty and D'Ambrosio (2010), Foster and Wolfson (2010) and Chakravarty (2009, 2015).

[6] In the literature on polarization, bipolarization is distinguished 'multipolar' polarization where the population is partitioned into multiple significantly sized subgroups and each subgroup is assumed to represent a pole. Esteban and Ray (1994) developed axiomatic formulation of a 'multipolar' polarization metric. For a systematic comparison of different polarization indices, see Esteban and Ray (2012), Chakravarty (2009, 2015) and Duclos and Taptué (2015). In an interesting contribution, Amiel et al. (2010) used a questionnaire-experimental approach to investigate whether people's perceptins on different notions of polarization are consistent with the corresponding key axioms.

We can now formally state the two basic postulates of bipolarization.

Increased Spread (IS) : For $x, y \in D^n_{++}$, where $m(x) = m(y)$ if any one of the following relations holds: (i) $x_+ = y_+$, $y_- \geq x_-$, (ii) $x_- = y_-$, $x_+ \geq y_+$, (iii) $y_- \geq x_-$, $x_+ \geq y_+$, then $P(x) \geq P(y)$.

Increased Bipolarity (IB): For $x, y \in D^n_{++}$, where $m(x) = m(y)$ if any one of the following relations holds: (i) $x_- E\, y_-$, $x_+ = y_+$, (ii) $x_+ E\, y_+$, $x_- = y_-$, (iii) $x_- E\, y_-$, $x_+ E\, y_+$, then $P(x) \geq P(y)$.

A simple example of a bipolarization index that satisfies IS and IB is

$$P_c(x) = \frac{\left(\frac{1}{n}\sum_{i=1}^{n}\left|m(x) - x_i\right|^c\right)^{\frac{1}{c}}}{m(x)}, \tag{10}$$

where $x \in D^n_{++}$ is arbitrary and $0 < c \leq 1$ is a parameter (see Chakravarty 2015). The constraint $0 < c < 1$ ensures that P_c satisfies IS and IB. P_c is simply the mean of order $0 < c < 1$ of the absolute values of the deviations of individual incomes from the median, normalized by the median itself. A reduction in the value of c over the interval $(0,1)$ increases P_c by a larger amount under a rank preserving progressive transfer on the either side of the median. The standard P_c coincides with the relative mean deviation about the median in the extreme case $c = 1$. On the other hand, as $c \to 0$, $P_c \to \frac{\prod_{i=1}^{n}\left|m(x) - x_i\right|^{\frac{1}{n}}}{m(x)}$, which becomes 0 if $x_i = m(x)$ for some i.

3. Background and Motivations

A taxation method F is a continuous function defined on the set D^1_{++} taking values in R_+. Formally, $F : D^1_{++} \to R_+$. For any income z, $F(z)$ is the tax liability of the person with income z. We do not restrict attention on positive taxes; a person's tax liability may be zero. We assume, however, that for any $x \in D^n_{++}$ the society's total tax collection $T = \sum_{i=1}^{n} t_i$ is positive, where $t_i = F(x_i)$. This ensures that there is at least one person in the society who has a positive tax burden. For any given $x \in D^n_{++}$, we write $t = (t_1, t_2, \ldots\ldots, t_n)$ for the associated profile of taxes. We denote the post-tax income distribution by $y = (x - t)$, where each post-tax income $y_i = (x_i - t_i)$ is non-negative.

The taxation method F is said satisfy the average progressivity taxation (APT) principle if $\frac{t_i}{x_i}$ is non-decreasing in x_i. It satisfies the minimally progressive taxation (MPT) rule if t_i is non-decreasing in x_i. The taxation method F is said to follow the incentive preservation (IP) property if $(x_i - t_i)$ is non-decreasing in x_i. In order to look at inequality implications of the two above notions of taxation, we now formally define Lorenz and absolute Lorenz dominations.

The post-tax income distribution y is said to Lorenz dominate the associated pre-tax income distribution x $(y \geq_L x, \textit{ for short})$ if $L(y,p) \geq L(x,p)$ for all $p \in [0,1]$. In words, the Lorenz superiority of y over x demands that the Lorenz curve of y lies nowhere below that of x. Since $y \geq_L x$ is equivalent to the condition that y is not more unequally distributed than x by all S-convex relative inequality indices, we refer to this condition as uniform relative equalization (URE).

The ordinate of the absolute Lorenz curve $AL(x,p)$ of the distribution $x \in D^n_{++}$ at the population fraction $\frac{j}{n}, 1 \leq j \leq n,$ is given by $\frac{\sum_{i=1}^{j}(x_i - \mu(x))}{n}$, the population size normalized cumulative income deviations of the first j persons from the mean. Then the absolute Lorenz curve $AL(x,p)$ of x is defined by setting $AL(x,0) = 0$ and

$$AL\left(x,\frac{j+\alpha}{n}\right)=(1-\alpha)AL\left(x,\frac{j}{n}\right)+\alpha AL\left(x,\frac{j+1}{n}\right) \qquad (11)$$

for all $\alpha \in [0,1]$.

The post-tax income distribution y is said to absolute Lorenz dominate the associated pre-tax income distribution x $\left(y \geq_{AL} x, \textit{for short}\right)$ if $AL(y,p) \geq AL(x,p)$ for all $p \in [0,1]$. Since $y \geq_{AL} x$ is equivalent to the condition that y is not more unequally distributed than x by all S-convex absolute inequality indices, following Moyes (1988), we refer to this condition as uniform absolute equalization (UAE).

We are now in a position to state the following progressivity results formally.

Theorem 1(Eichhorn et al., 1984): Properties APT and IP hold together if and only if the property URE holds.

Theorem 2(Moyes, 1988): Properties MPT and IP hold together if and only if the property UAE holds.

In order to relate progressivity to bipolarizaion, we need to define the relative bipolarization curve of an income distribution. The ordinate of the relative bipolarization curve of the distribution $x \in D_{++}^{n}$ at the population fraction $\frac{j}{n}$ is the sum of the income shortfalls $\sum_{j \leq i < \bar{n}} \frac{\left(m(x)-x_i\right)}{nm(x)}$, normalized by the factor $nm(x)$, of the first j individuals whose incomes are below the median, from the median itself, where $1 \leq j < \bar{n}$ $,\bar{n}=\frac{n}{2}$. Similarly, for incomes not below the median the ordinate at the population proportion $\frac{j}{n}$ is the sum of the normalized excesses $\sum_{\bar{n} \leq i \leq j} \frac{\left(x_i - m(x)\right)}{nm(x)}$ over the median itself, where $\bar{n} \leq j \leq n$. The relative bipolarization curve $RB(x,p)$ of $x \in D_{++}^{n}$ is then defined as

$$RB\left(x,\frac{j+\alpha}{n}\right)=(1-\alpha)RB\left(x,\frac{j}{n}\right)+\alpha RB\left(x,\frac{j+1}{n}\right) \qquad (12)$$

for all $\alpha \in [0,1]$ and $1 \le j \le (n-1)$, where $RB(x,0) = 0$.

The pre-tax income distribution x is said to bipolarization dominate the associated post-tax income distribution y $\left(x \ge_{BP} y, \, for\ short\right)$ if $RB(x,p) \ge RB(y,p)$ for all $p \in [0,1]$. Alternatively, we can say y depolarization dominates x. In other words, taxes are depolarizing. Since $x \ge_{BP} y$ is also equivalent to the condition that y is regarded as not more bipolarized than x by all relative bipolarization indices that satisfy anonymity, IS and IB, we refer to this condition as uniform relative depolarization (URD)[7].

The following theorem that shows the relationship between $\ge_L$ and $\ge_{BP}$ can now be stated:

Theorem 3 (Carbonell –Nicolau and Llavador, 2021): Property URE holds if and only if property URD holds

We close this section with a brief analytical discussion on the IMT principle introduced by Chakravarty and Sarkar (2022). Given that a total tax of size T is to be collected from the individuals in the society in which income distribution is $x \in D^n_{++}$, let $q_i = \frac{1}{n-i+1} \sum_{j=i}^{n} x_j$, $i = 1,2,......,n$, be the partial means of the right tails of x. Next, define k as

$$k = \min\left\{i \in \{1,2,....,n\} : (n-i+1)(q_i - x_i) \le T\right\}. \tag{13}$$

Now, define $y \in D^n_{++}$ to be the distribution $\Lambda(x,T)$ in the following way:

$$y_i = \begin{cases} x_i \ \ for\ 1 \le i \le (k-1), \\ q_k - \dfrac{T}{n-k+1} \ \ for\ k \le i \le n. \end{cases} \tag{14}$$

Observe that in the polar case if the tax size T equals 0, then $\Lambda(x,T) = x$.

The following illustrative example is taken from Chakravarty and Sarkar (2022).

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[7] See Chakravarty (2009, Theorem 4.3) and Chakravarty (2015, Theorem 2.2). If the two distributions have the same median, then this ordering turns out to be equivalent to the Foster-Wolfson (2010) polarization ordering-or its equivalent formulation suggested by Wang and Tsui (2000)-and the ordering $\ge_0$ considered in Theorem 1 of Bossert and Schworm (2008).

Example 1: Suppose $x=(10,15,20,30)$ and $T=20$. Then $q_1=\frac{75}{4}$, $q_2=\frac{65}{3}$, $q_3=25$ and $q_4=30$. Hence $k=2$. Consequently, the post-tax distribution $y=\left(y_1,y_2,y_3,y_4\right)$ comes to be $y=(10,15,15,15)$. Hence the inequality minimizing tax schedule is given by $t=(0,0,5,15)$.

In view of Proposition 3 and Corollary 2 of Chakravarty and Sarkar (2022) we can now state the following:

Theorem 4: Given any pre-tax income distribution $x \in D^n_{++}$ and a tax size $T>0$, let $\Lambda(x,T)$ be defined using (14). Then the scheme $x-\Lambda(x,T)$ is an IMT scheme for any inequality index that satisfies anonymity and the Pigou-Dalton transfer principle.

Since Theorem 4 relies on inequality metrics that satisfy only anonymity and the transfer postulates, it holds for a large class of inequality standards. Mention-worthy among these are the Atkinson (1970) (relative) index, the (relative) generalized entropy family which contains the two Theil (1967, 1972) indices and half the squared coefficient of variation as particular cases, the Kolm (1976) (absolute) index, the variance, the Gini and the Bonferroni indices and their absolute sisters. It as well holds for inequality standards satisfying the Bossert-Pfingsten (1990) intermediate inequality invaration. (Cowell's (2016) survey provides detailed discussion on different inequality metrics.)

It becomes certainly worthy to analyze when an inequality minimizing tax scheme makes the resulting post-tax distribution perfectly equal. The following theorem specifies the necessary and sufficient condition in this context.

Theorem 5: Given any pre-tax income distribution $x \in D^n_{++}$ and a tax size $T>0$, let $y \in D^n_{++}$ be obtained from x and T, as given by (14).Then $I(y)=0$ if and only if $T \geq \sum_{i=1}^{n} x_i - nx_1$.

Proof: Suppose $I(y)=0$. Since taxation does not increase income it follows that $x_1 \geq y_1 = y_2 = ...= y_n$, where equality of the post-tax incomes is ensured by the normalization condition. Note that $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} x_i - T$ and so $\sum_{i=1}^{n} y_i \le n x_1$ implies that $T \ge \sum_{i=1}^{n} x_i - n x_1$. Conversely, suppose $T \ge \sum_{i=1}^{n} x_i - n x_1$. From the definition of k in (14) it follows that $k = 1$. Then from the definition of y it follows that $y_1 = y_2 = \ldots\ldots\ldots = y_n$, which in view of the normalization condition establishes that $I(y) = 0$. Δ

The following corollary of Theorem 5 specifies the necessary and sufficient conditions for the post-tax inequality under an IMT policy to be positive.

Corollary 6: Given any pre-tax income distribution $x \in D_{++}^{n}$ and a tax size $T > 0$, let $y \in D_{++}^{n}$ be obtained from x and T, as given by (14).Then the following statements are true.

(a)Given $I(y) > 0$ it is necessarily true that $I(x) > 0$ and $T < \sum_{i=1}^{n} x_i - n x_1$.

(b) For $I(y) > 0$ to hold it is sufficient that $I(x) > 0$ and $T < \sum_{i=1}^{n} x_i - n x_1$.

This corollary stipulates that the post-tax incomes resulting from an IMT scheme are unequal if and only if the pre-tax incomes are unequal and the total tax size is less than aggregate excesses of individual incomes over the minimum income.

Given that the inequality minimizing taxation principle is quite appealing from an egalitarian perspective, a natural question that arises here how is it related to the average progressive, minimally progressive and bipolarization reducing taxation rules? We investigate these issues in the next section of the paper.

4. Comparative Analysis

For results to be presented in this section we assume at the outset that the total taxes collected under different tax schemes are the same. Moyes(1988) demonstrated that APT and MPT are logically independent in the sense that there exist taxation schemes that satisfy (i) both APT and MPT, (ii) APT but not MPT, (iii) MPT but not APT, and (iv) neither APT and MPT. In order to establish their relationships with IMT, let us consider the pre-tax income distribution

$x=(10,15,20,30)$ taken in Example1.The tax schedule $t'=(0,3,4,13)$ corresponding to the distribution $x=(10,15,20,30)$ is both average progressive and minimally progressive, but not inequality minimizing although it collects the same amount of total revenue as the inequality minimizing tax profile $(0,0,5,15)$. This example clearly demonstrates that neither average progressivity (hence depolarization) nor minimally progressivity of a tax function is sufficient for inequality minimizing taxation. The converse is shown to be true in the following theorem.

Theorem 7: IMT implies APT and MPT.

Proof: Given $x \in D_{++}^{n}$ and $T>0$, let k be defined from x and T, as in (13). Next, suppose that y is obtained from x and T using (14). Let t be derived as $t=x-y=(t_1,t_2,....,t_n)$.

Then for $i=1,2,\ldots,k-1$, $y_i=x_i$, and for $i=k,\ldots,n$, $y_i=x_k-\frac{T}{n-k+1}$, So for $i=1,2,\ldots,k-1$, $t_i=0$, and for $i=k,\ldots,n$, $t_i=x_i-\delta$, where $\delta=x_k+\frac{T}{n-k+1}$. Since $t_1=t_2=t_{k-1}=0$, it is sufficient to consider $i\geq k$.

For $k\leq i\leq n-1, t_i\leq t_{i+1}$ if and only if $x_i-\delta\leq x_{i+1}-\delta$. Since x_i's are monotone non-decreasing, the condition $x_i-\delta\leq x_{i+1}-\delta$ holds. This proves that IMT implies MPT.

For $k\leq i\leq n$, $\frac{t_i}{x_i}=1-\frac{\delta}{x_i}$ and so $\frac{t_i}{x_i}\leq\frac{t_{i+1}}{x_{i+1}}$ if and only if $1-\frac{\delta}{x_i}\leq 1-\frac{\delta}{x_{i+1}}$, that is, if and only if $\frac{1}{x_{i+1}}\leq\frac{1}{x_i}$. Again, since x_i's are monotone non-decreasing, the condition $\frac{1}{x_{i+1}}\leq\frac{1}{x_i}$ holds. This demonstrates that IMT implies APT. Δ

The following proposition shows that an inequality minimizing taxation scheme is depolarizing.

Proposition 8: IMT implies that the underlying taxes are depolarizing, that is, the underlying post-tax income distribution depolarization dominates the pre-tax distribution.

Proof: By the equivalence of the Lorenz and depolarization dominances of the post-tax distribution over the corresponding pre-tax distribution we know the specific taxation schedule satisfies APT if and only if it does not raise bipolarization (Theorems 1 and 3). Now, Theorem 5 shows that IMT implies APT. This shows that an inequality minimizing taxation program is depolarizing. Δ

To show that the converse of Proposition 8 is untrue, let us consider once again the pre-tax distribution $x = (10,15,20,30)$ of Example 1. It is easy to verify that the bipolarization curve of the inequality minimizing post-tax distribution $(10,15,15,15)$ lies nowhere above that of the post-tax distribution $(10,12,16,17)$ associated with the taxation program $x = (0,3,4,13)$ satisfying APT.

The following Proposition establishes that inequality minimizing taxation minimizes bipolarization as well.

Proposition 9: IMT implies that the underlying taxes minimize bipolarization also.

Proof: Let w be the IMT obtained from x and T and let z be any other post-tax distribution satisfying IP which is also obtained from x and T. Then $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} z_i$. Further, the proof of Theorem 1 of Chakravarty-Sarkar (2022) shows that w Lorenz dominates z. From Theorem 3 it follows that z bipolarization dominates w. Hence the bipolarization of w is at most that of z. By Proposition 8 taxes underlying IMT are depolarizing. Hence IMT also minimizes bipolarization. Δ

Since depolarizing taxes need not be inequality minimizing, the converse of Proposition 9 is not true.

In order to make a systematic comparison between welfare levels of the post-tax distributions associated with different taxation programs, let us assume at the outset that all taxation programs are incentive preserving. Given the pre-tax income profile $x \in D_{++}^{n}$ and tax size $T > 0$, let the taxation schedules $t^{AP} = \left(t_1^{AP}, t_2^{AP}, \ldots, t_n^{AP}\right)$ and $t^{MP} = \left(t_1^{MP}, t_2^{MP}, \ldots, t_n^{MP}\right)$ associated with x be respectively average progressive and minimally progressive. We denote the corresponding post-tax income distributions by $y^{AP} = \left(y_1^{AP}, y_2^{AP}, \ldots, y_n^{AP}\right)$ and $y^{MP} = \left(y_1^{MP}, y_2^{MP}, \ldots, y_n^{MP}\right)$ respectively.Next, for x and $T > 0$, suppose that $y^{IM} = \left(y_1^{IM}, y_2^{IM}, ..., y_n^{IM} \right)$ is obtained using (14). We write $t^{IM} = \left(t_1^{IM}, t_2^{IM}, ..., t_n^{IM} \right)$ for the corresponding distribution of taxes.

If the social evaluation is done with respect to the Gini welfare function and a policymaker prefers IMT to APT, then the resulting size of welfare gain is given by $W_G\left(y^{IM} \right) - W_G\left(y^{AP} \right)$.For the Bonferroni welfare function this size turns out to be $W_B\left(y^{IM} \right) - W_B\left(y^{AP} \right)$. Likewise, when one prefers to choose IMT instead of MPT, these sizes become respectively $W_G\left(y^{IM} \right) - W_G\left(y^{MP} \right)$ and $W_B\left(y^{IM} \right) - W_B\left(y^{MP} \right)$.

We may now illustrate this using the example considered above. For the pre-tax income distribution $x = (10,15,20,30)$, the values of the Gini and Bonferroni welfare functions for the post-tax distribution $y^{IM} = (10,15,15,15)$ resulting from the inequality minimizing tax profile $t^{IM} = (0,0,5,15)$ are given respectively by $W_G(10,15,15,15) = \frac{205}{16}$ and $W_B(10,15,15,15) = \frac{595}{48}$. These values for the post-tax income distribution $(10,12,16,17)$ corresponding to the average and minimally progressive tax profile $t' = (0,3,4,13)$ are respectively $W_G(10,12,16,17) = \frac{195}{16}$ and $W_B(10,12,16,17) = \frac{569}{48}$. Consequently, the sizes of the Gini and Bonferroni welfare gains here become $\frac{10}{16}$ and $\frac{26}{48}$ respectively.

5. An Empirical Illustration

We compare the IMT policy with the existing taxation structure in India using the income data collected by the Center for Monitoring Indian Economy (CMIE). The CMIE's Consumer Pyramids Household Survey (CPHS) is an ongoing nationally representative longitudinal survey of Indian households. The sample selection for CPHS is based on the Census of India, 2011. The stratification for this survey is done using homogenous regions which are created to represent similar agro-climatic conditions (Vyas, 2020). These homogenous regions are further divided into five strata representing villages and various town sizes. The towns are over-represented to capture the variety present in the urban sector. Although this is a panel survey, the sample changes slightly in each round and the number of households/individuals surveyed is not exactly the same across rounds. In the May-August 2018 wave, the sample consists of 172,365 households from 585 Indian districts (Vyas, 2020).

The dataset reports the individual incomes from various sources. The estimation of the pre-tax and post-tax inequality in individual incomes[8] is based on the data for financial year 2018-19[9] taken from CPHS waves 13-16. Moreover, we focus on the prime working age group, 15-59 years. The selection of this age-group is to avoid children and senior citizens[10]. The dataset provides monthly incomes for all the individuals. Each round of the survey is for four months and the individuals' incomes for all the four months are collected at the same time. These incomes, as self-reported by individuals, are considered as the pre-tax incomes for estimation of inequality. Since this sample has an urban bias, we use the sampling weights to get the national estimates of incomes and inequality.

As mentioned earlier, the panel changes slightly for each round and the information is not available for all the individuals for all the months in 2018-19 (Vyas, 2020b). We consider only those individuals (94.22 percent in the relevant age group with positive incomes) for whom the information are available for the entire year. With all the above mentioned considerations, the final data set includes 2, 13,123 individuals with positive incomes in the 15-59 age group.

Table 1: Taxation Structure for Indian Individuals below 60 years, 2018-19

Income	Tax rate	Average Tax Rate under Current Taxation Structure	Average Tax Rate under IMT
Up to 2,50,000	Nil	0%	0%
2,50,001-5,00,000	5%	1.22%	0%
5,00,001-10,00,000	20%	5.65%	4.31%
Above 10,00,000	30%	13.87%	44.05%

Note: The average tax rates are estimated for individuals with total pre-tax incomes in the given tax slab based on various waves of CMIE Consumer Pyramids Survey.

[8] Here, for illustrative purpose, we focus on the inequality in individual income and ignore the combined income for all household members.
[9] The 2018-19 is the last pre-pandemic financial year. Thus, we use this year to avoid abnormalities due to the pandemic.
[10] In India, the tax slab for senior citizen differs from the one for below 60 years. The age restriction maintains the progressivity of taxes.

We compare the pre-tax income inequality with the inequality under two different taxation policies: (1) existing taxation structure in India and (2) IMT policy. The comparison relies on the assumption that the total tax considered for generating the inequality minimizing taxation is the same as the actual total tax deduction. Under the existing taxation structure in India, the tax rates applicable for various income levels in 2018-19 are reported in Table 1. We estimate the post-tax incomes based on this taxation structure for the relevant age-group. Next, we estimate the post-tax incomes under the IMT policy using equation (13) and (14).

From Table 1 we note that the average tax rates are non-decreasing under both the taxation structures. Hence, by Theorem 1, the after-tax distribution cannot be regarded as more unequal than the before-tax distribution by all S-convex relative inequality indices. By Theorem 3 we can also claim that the tax rates under the current taxation scheme, as reported in Table 1, did not increase bipolarization. In view of Proposition 8 this claim as well holds for the IMT policy. Consequently, both the 2018-19 Indian taxation structure and IMT led to an improvement in the position of the middle income group among the tax payers (below aged 60) in the economy.

In addition to simply comparing before- and after-tax inequality levels, it also becomes innovative to examine the extent of effective progression caused by a taxation structure. (The term ‘effective progression’ was introduced by Musgrave and Thin (1948). An index of effective progression is a summary measure of shifts in the distribution of income toward equality generated by the taxation system as a whole.) Given that the average tax rates are non-decreasing under both the schemes, the extents of effective progression under the two schemes should be positive.

Liu (1985) suggested the use of the difference between the Gini indices for pre-and post-tax incomes as a measure of effective progression:

$$P_L(x,t) = I_G(x) - I_G(y). \tag{15}$$

This index is increasingly related to the classical Musgrave-Thin (1948) index of effective progression

$$P_{MT}(x,t) = \frac{1 - I_G(y)}{1 - I_G(x)}, \tag{16}$$

through the transformation $P_{MT}(x,t)=1+\frac{P_L(x,t)}{1-I_G(x)}$. Given a pre-tax income distribution, hence for a fixed value of $I_G(x)$, an increase in P_L is equivalent to an increase in P_{MT} and vice-versa. Therefore, for given pre-tax incomes, these two measures will rank the post-tax distributions associated with two taxation schemes in the same way in terms of effective progression.

We observe that under the current taxation scheme the Gini coefficient reduces from 0.4633 for pre-tax incomes to 0.4584 for post-tax incomes[11] (Table 2), which in turn generates a value of 0.0049 for P_L. This is an implication of a low average tax rate (among all taxpayers below 60) of 2.33 percent. It may be worthwhile to compare our findings with those of Datta et al. (2021) although data sources are different. The authors analyzed the redistributive effect of the Indian taxation system using Indian Income Tax Department data for the period 2011-2018. They noted that the value of P_L has been low at around 0.05 over the period under consideration[12]. The average tax rate (among all taxpayers) over the period has been more or less around 9-10%. Thus, even though data sources are different the findings in the two situations are similar.

The Gini coefficient under IMT reduces marginally further from 0.4584 to 0.4582, showing a value 0.0051 of P_L, indicating that there is a minor improvement in effective progression when one follows the IMT policy instead of the current taxation structure. Since IMT implies bipolarization minimization (Proposition 9), this improvement in effective progression can also be attributed to a taxation policy raising taxes in a bipolarization minimizing way. The positive value of the after-tax Gini index under the IMT policy ensures that the CMIE data set used here verifies the necessary condition (a) stipulated in Corollary 6, showing a nice application of the corollary to real life data.

We also compare the welfare gains, as given by equation (8), for the two taxation policies. Table 2 depicts higher social welfare under IMT as compared to the current tax policy.

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[11] Here, while estimating the post-tax inequality, we ignore the deductions as the relevant information is not available in the dataset. If we allow for the deductions, then the inequality reduction will be even lower.

[12] It may be noted here that Datta et al. (2021) estimate the progressivity for all tax assesses, whereas we measure it among all individuals (irrespective of whether they file the tax returns) in the given age group of 15-59.

Table 2: Welfare Gains under Different Tax Policies

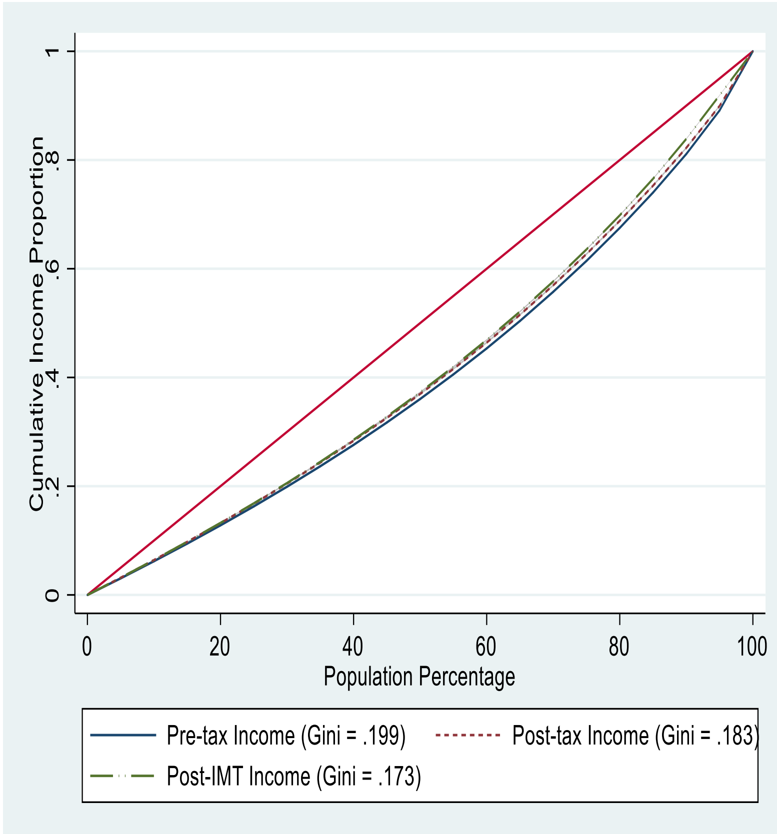
Taxation Policy	Gini Coefficient	Social Welfare
Pre-tax	0.4633	57994.08
Post-Tax	0.4584	57968.39
Post-IMT	0.4582	57991.25

Source: Estimation based on CMIE Consumer Pyramids Survey, various waves

We now look at implications of the two taxations principles in terms of equity only on the tax payers. Since the proportion of the taxpayers is very less in India, the observed magnitude of effect on inequality and welfare is also small. The comparison of the inequality among the taxpayers[13] shows a greater reduction in inequality (Figure 1). The reason behind this is that when zero-tax payers are included, for a sizeable proportion of population there are no differences between pre-and post-tax incomes. As a result under each of the two taxation scheme the post-tax inequality does not move away much from its pre-tax counterpart. All the claims we have made earlier in the section, as applications of Theorems 1 and 3; Corollary 6 and propositions 8 and 9, remain valid here as well. The pre-tax Gini coefficient for the taxpayers is 0.199 and this reduces to 0.183 under the existing taxation structure. If the IMT is implemented then the Gini coefficient further reduces to 0.173. This in turn shows that the values of P_L under the current taxation scheme and the IMT structure are respectively 0.016 and 0.026, which are much higher than the corresponding figures (0.0049 and 0.0051 respectively) when zero-tax are included. In fact, the percentage increase in progression is much higher under IMT (400%) than that under the current system (227%). Figure 1 shows a clear Lorenz dominance of after-tax incomes under IMT over that under the current structure. Thus, the empirical findings are consistent with the theory where the inequality is minimum under IMT and the social welfare is higher as compared to India's existing progressive taxation system.

[13] The CPHS data only reports the individual's income and the tax liability is calculated using the tax slabs given in Table 1. The dataset does not provide the information on whether these individuals have actually paid the tax.

Figure 1: Pre-tax, Post-tax and Post-IMT Lorenz Curves for Taxpayers



Source: Estimation based on CMIE Consumer Pyramids Survey, various waves

In this context, it is also enlightening to examine the no-tax thresholds and the proportion of individuals taxed under these two policies. Suppose that the tax collection target is T.Given pre-tax distribution x and T, we can determine the post-tax distribution y underlying the IMT scheme. For y, suppose θ is the no-tax threshold. We may then say that in the actual taxation scheme, the no-tax threshold should also be a value close to θ. This provides a formal justification for determining the no-tax threshold of an actual taxation scheme. Presently, the no-tax threshold is determined in an ad-hoc manner. The present no-tax threshold is at ₹ 250,000. However, under the IMT structure, this threshold is much higher at ₹ 650,500.

We may define a measure of low-income sensitivity of a tax scheme in the following manner. For given pre-tax distribution $x \in D^n_{++}$ and tax size $T > 0$, recall the construction of y^{IM} using (14). Let y be the actual post-tax income distribution. Let r_1 be the no-tax threshold of y^{IM} and p_1 be the percentage of the population whose incomes are below r_1. Similarly, let r_2 be the no-tax threshold of y and p_2 be the percentage of the population whose incomes are below r_2. Then we may define the low-income sensitivity of y as $\delta = \frac{p_2}{p_1}$. In general, p_1 would be at least as much as p_2 so that the positive quantity δ is at most 1. A low value of δ suggests that a lower proportion of people were not taxed in comparison to IMT, while a high value of δ close to 1 suggests that the proportion of people who were taxed is almost that required by IMT. Therefore, from policy perspective a society may prefer a high value of δ. The empirical results show that under the current taxation structure, 7.38 per cent of working age individuals are taxed. Under the IMT policy, the same tax revenue is collected from only 0.6 percent individuals in the relevant age group. As a result, the low-income sensitivity of the Indian tax structure is δ =0.9318.

6. Conclusions

The well-known taxation principle 'average progressivity' results in non-increasingness of inequality through reduction of relative income differences (Eichhorn et al., 1984). In comparison with this, the 'minimally progressive' taxation rule leads to non-rising inequality by blowing off absolute income gaps (Moyes, 1988). This paper rigorously demonstrates that these two criteria of tax progressiveness that are explicitly dependent on definite concepts of inequality invariances are implied by the recently introduced 'inequality minimizing' taxation principle (Chakravarty and Sarkar, 2022), which does not rely on any notion of inequality invariance. It is also shown that inequality minimizing taxation leads to depolarization of income distribution. In all cases the reverse implications are shown to be untrue.

Data Availability

Data will be made available on request.

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