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Rupayan Pal and Emmanuel Petrakis

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Email(corresponding author): empetrak@eco.uc3m.es

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Keywords: Cross-ownership, passive shares, strategic substitutes and complements, divestment incentives, market competition.

JEL Code: L13, L41, L2, D43

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Rupayan Pal[1] and Emmanuel Petrakis[2]

[1]Indira Gandhi Institute of Development Research (IGIDR), Mumbai, India

[2]Departamento de Economía, Universidad Carlos III de Madrid, Spain

Abstract: This paper shows that in a duopoly a firm has no incentives to divest its passive shares in its rival when firms' strategies are strategic complements. This holds independently whether goods are substitutes or complements and whether firms engage in simultaneous or sequential move product market competition. However, if firms' strategies are strategic substitutes and are engaged in simultaneous move competition, it is optimal for both firms to fully divest their shares in their rivals under a private placement mechanism via independent intermediaries or under competitive bidding. Yet, in the sequential move game only the follower has such incentives. Notably, under a private placement mechanism via a common intermediary, there are circumstances under which there are partial or no firms' divestment incentives, highlighting that the divestment mechanism employed by firms may have a crucial role on their divestment incentives.

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Corresponding Author and Address: Emmanuel Petrakis.
Emails: (R. Pal) rupayan@igidr.ac.in, rupayanpal@gmail.com. (E. Petrakis) empetrak@eco.uc3m.es

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1. Introduction

The phenomenon of cross-holdings among competing firms in a sector is widespread and on the rise in recent years. For instance, Keiretsu is a large cluster of business groups where member companies hold a fraction of shares in other companies (Grabowiecki, 2006). Heim et al. (2022) report 10,699 cases of minority acquisitions in rival firms across 63 countries between 1990 and 2013.[1] Nain and Wang (2018) report 1068 minority stake acquisitions among rival firms in the US manufacturing sector between 1980 and 2010. He and Huang (2017) documents 50% increase in the fraction of U.S. public firms that are cross-held during 1980 and 2014. Gilo et al. (2006) and Gilo (2000) report several instances of passive cross-ownership across different sectors. For instance, in the Automobile sector, Toyota bought a 4.94% stake in Suzuki, while Suzuki bought 48-billion-yen worth of shares in Toyota in 2019 (Shiraki and Yamazaki, 2019). In 2018, Renault owned 43% of Nissan, while Nissan had a 15% share in Renault; also, Volkswagen and Suzuki held shares in each other (19.89% and 2.5%) from 2011 to 2015 (Hariskos et al, 2022).[2]

These observations made researchers, policymakers, and competition authorities to delve into the potential implications that firms' cross-holdings may have on the performance of markets, consumer and social welfare. Both the theoretical and empirical literature have pointed out that cross-holdings among rival firms in a sector could have substantial anticompetitive effects, harming the consumers and the society.

A natural question that arises is whether these observed ownership structures will turn out to be stable in the long run. Do firms holding passive shares in their rivals have incentives to divest

[1] 43% of these acquisitions had a stake size of 25–50% in the rival firm, 36% had a stake size of 10–25% and 21% of the acquisitions had a stake size of 0–10%.

[2] For the Financial sector, Trivieri (2007) reports that cross-ownership was widely observed in the Italian banking industry in the years 1996-2000. Dietzenbacher et al. (2000) documents existence of cross-ownership in the Dutch financial sector. According to Azar et al. (2022) and Barth et al. (2022) cross-ownership is a prevalent phenomenon in the U.S. Banking Sector. In Information Technology, Microsoft acquired almost 7% of the nonvoting stock of its rival Apple in August 1997, and in June 1999 it took a 10% stake in its rival Inprise/Borland Corp. (Ezrachi and Gilo, 2006). In Consumer Goods, Gillette acquired 22.9% of the nonvoting stock and approximately 13.6% of the debt of one of its largest rivals Wilkinson Sword (Ezrachi and Gilo, 2006). For the Resource sector, Dai et al. (2022) reports that (i) BP holds a 19.75% stake in the Russian oil giant Rosneft; (ii) the Mexican state-owned petroleum company Pemex holds a 9.3% stake in the Spanish oil giant Repsol; and (iii) China's state-owned Sinopec holds a 30% stake in Petrogal Brasil, and 40% in Repsol YPF Brasil, respectively.

them by selling their shares in rival firms to outside investors; and if so, under what conditions? If firms have divestment incentives, the harm to consumers and the society will be only in the short run and policy measures to correct for potential market inefficiencies may not be justifiable. A subsequent question that demands an answer is whether firms have similar divestment incentives under alternative divestment mechanisms.

In practice, firms use alternative divestment mechanisms. Public offering and private placement are the two dominant mechanisms being used by firms in recent decades. Public offering refers to sale of equity shares by listed firms to the public through open market, whereas private placement refers to sale of shares by listed or unlisted firms to a limited number of pre-selected investors. The latter may be carried out through intermediaries (placement agents) or directly through negotiation between the issuer and investor(s) and are subject to less stringent regulations compared to public offerings. In initial public offering (IPO), initial share price is generally determined through negotiation between the issuing firm and underwriting investment banks. However, firms can use alternative mechanisms, e.g. competitive bidding, to offer IPO. In the case of competitive bidding, the share price of IPO is determined by the market through investors' bids, wherein underwriters' price setting power is almost non-existent. While most IPOs go through the traditional mechanism, some firms have opted for the unconventional mechanism of competitive bidding for their IPOs. For example, Google raised USD 1.67 billion in August 2004 by using Dutch auction process for its IPOs (Choo, 2005). Notably, divestment through private placement has become increasingly popular over time (Gao et al., 2022; Lerner et al., 2015; Foerster and Karolyi, 2000).[3]

To address the above questions, we consider a reduced form simultaneous move duopoly market game in which firms' goods are either substitutes or complements and firms' strategies are either strategic substitutes or strategic complements. Each firm owns an arbitrary number of (non-

[3] For example, private equity and venture capital funds invested over USD 11 billion in the telecom sector in India in the year 2020 alone (Praxis Global Alliance, 2021). In 2016, total amount of funds raised by A-share listed companies in China by private placement was more than ten times of the amount raised by initial public offerings (Song et al., 2023; Dong et al., 2020). Private placements of stocks have grown substantially compared to public offerings in the U.S. following the adoption of Rule 144A in 1990 (Bolton et al., 2016; Zingales, 2009; Wruck and Wu, 2009). Using the U.S. Census measures of industry concentration data, Ali et al. (2014) empirically demonstrate that firms in more concentrated industries prefer to raise funds through private placements compared to public offerings, presumably because private placement helps to avoid potential leakage of information to rival firms unlike public offerings.

controlling) passive shares in its rival and its manager maximizes the firm's accounting profits. As is standard, the latter are equal to the firm's operating profits multiplied by the shares not owned by its rival plus the rival's operating profits multiplied by the shares owned in the rival firm. In this setup, we examine the firms' incentives to divest their passive shares in their rivals before engaging in market competition. We consider alternative divestment mechanisms. A private placement mechanism via either independent intermediaries or a common intermediary, and a competitive bidding mechanism such as a Dutch auction.

Our main findings are as follows. Independently whether firms' goods are substitutes or complements, if the firms' strategies are strategic complements, no firm has incentives to divest its passive shares in the rival. This result holds under all divestment mechanisms under consideration. This implies that for any given arbitrary ownership structure, we can proceed in analyzing its market and societal implications with no need to worry about the stability or not of such a structure.

Nevertheless, when firms' strategies are strategic substitutes and the divestment mechanism is either private placement via independent intermediaries or competitive bidding, firms always have incentives to fully divest their passive shares in their rivals, before engaging in product market competition. This holds independently whether goods are substitutes or complements. This finding raises a word of caution when we analyze the market and societal implications of a given ownership structure as it questions its initial stability. Interestingly, the opposite occurs when the divestment mechanism is private placement via a common intermediary, *unless* the ownership structure and/or the market characteristics are substantially asymmetric. In the latter case, firms may or may not have divestment incentives depending in all market specificities. This finding highlights the crucial role that the divestment mechanism used by firms may have on the firms' divestment incentives.

We extend our analysis in a sequential move reduced form duopolistic market game and show that strategic complementarity of firms' strategies again prevents firms from divesting their passive shares in rivals. Nevertheless, independently whether goods are substitutes or complements, when firms' strategies are strategic substitutes, the leader has no incentives to divest its passive shares in the follower, while the follower has incentives to fully divest its shares in the leader. This finding indicates that divestment incentives may differ in industries in which firms make simultaneous strategic decisions than in those in which firms decide sequentially.

Related Literature

Our paper belongs to the strand of the literature that examines the formation and the stability of ownership structures. A seminal paper in this literature is Reitman (1994). The author shows that if there are at least three firms in the market, firms have no incentives to form pairwise partial ownership agreements under Cournot competition, unlike as in the case of Bertrand competition. However, from the analysis of Reitman (1994) it follows that, if there is duopoly in the product market, it is jointly optimal for firms to form pairwise partial ownership agreements regardless of the mode of product market competition. Flath (1991) examines firms' incentives to acquire passive shares in their rivals. The author considers that share prices of firms are determined through an efficient market mechanism. Flath shows that cross-ownership is optimal for firms under Bertrand competition, but not under Cournot competition. However, cross ownership is jointly optimal under Cournot competition as well, implying that cross ownership may serve as an instrument for tacit collusion.[4]

The closest paper to ours is Stenbacka and Moer (2021). Under a simultaneous move product market competition in which firms' strategies are strategic substitutes, the authors show that firms have incentives to sell their passive shares in a rival firm to independent outside investors, implying that cross ownership in a Cournot duopoly is not stable.

Our paper contributes to this strand of literature by examining stability of cross ownership in a more general reduced form duopoly, which allows for firms' strategies to be either strategic substitutes or strategic complements and for firms' goods to be either substitutes or complements. Moreover, it considers both simultaneous and sequential move product market competition. We confirm Stenbacka and Moer's findings, but only if the divestment mechanism is private placement via independent intermediaries or competitive bidding. Yet, under private placement via a common intermediary, in contrast to Stenbacka and Moer, firms have no divestment incentives when the ownership structure and market characteristics are rather symmetric. Only if there are substantial

[4] In a different vein, Brito et al. (2014) consider the effect on consumers' welfare of a firm's partial ownership of its rival and compare the implications of alternative forms of divestiture. They focus on the conditions under which turning voting shares into nonvoting shares is preferable to selling the shares to the firm's current shareholders. They also show that selling the voting shares to a large independent shareholder is preferable to selling them to small shareholders.

asymmetries in these respects, one or both firms may have incentives to (partially or fully) divest their shares in the rival firm.

The rest of the paper is organized as follows. Section 2 describes our model along with its assumptions. Section 3 analyzes the firms' divestment incentives when they decide simultaneously their market strategies and the divestment mechanism is private placement via independent intermediaries. Section 4 extends the analysis to alternative divestment mechanisms. Section 5 analyzes a sequential move product market game under a private placement mechanism with independent intermediaries. Section 6 contains concluding remarks. All proofs are relegated in the Appendix.

2. The Model

Suppose that there are two firms, firm 1 and firm 2, operating in a product market. Let $g_i \in \mathbb{R}^+$ be the strategic variable of firm i, such that a higher value chosen for g_i indicates a "more aggressive play" by firm i. Each firm holds passive shares of its rival. Let $s_i \in (0, \frac{1}{2}]$ be the fraction of firm j's shares owned by firm i and let $\pi_i(g_i, g_j)$ denote firm i's operating profit. Firm i can resell $r_i \in [0, s_i]$ of its passive shares in firm j to an outside investor, I_i, for a fixed fee $F_i (\geq 0)$ via a take-it-or-leave-it offer. The outside investor buying from firm 1 is different from the outside investor buying from firm 2, i.e., $I_1 \neq I_2$.[5] Assume that firm i's manager maximizes its firm's accounting profit given by,

$$\Gamma_i(g_i, g_j, r_i, r_j) = \alpha_i F_i + \pi_i(g_i, g_j) + (s_i - \alpha_i r_i)\Gamma_j(g_i, g_j, r_i, r_j), \tag{1}$$

where $\alpha_i = 1$, if outside investor I_i accepts the offer from the manager of firm i; otherwise, $\alpha_i = 0$. From (1), we get:

$$\Gamma_i(g_i, g_j, r_i, r_j) = \frac{\alpha_i F_i + \pi_i(g_i, g_j) + (s_i - \alpha_i r_i)\left(\alpha_j F_j + \pi_j(g_i, g_j)\right)}{1 - (s_i - \alpha_i r_i)(s_j - \alpha_j r_j)} \tag{2}$$

[5] This divestment mechanism is equivalent to a *private placement* mechanism in which firms sell (part or all) of their shares in their rivals via independent intermediaries, M_i and M_j, to different outside investors, I_i and I_j. In subsection 3.2, we examine the case of a *private placement* divestment mechanism in which the two firms use a common intermediary (e.g., a bank) to sell their shares to outside investors.

Clearly, due to cross-holdings, the manager of firm i internalizes firm j's profit while choosing the level of g_i, and the extent of such internalization depends on the fraction of its *retained* shares $(s_i - \alpha_i r_i)$ in firm j.

To guarantee that a unique interior equilibrium exists and is stable in all the cases considered in this paper, we make the following assumption in the sequel.

Assumptions:

A1 $\Gamma_i(g_i, g_j)$ is a twice continuously differentiable function in g_i and g_j, for all $\alpha_i \in \{0, 1\}$, $s_i \in \left[0, \frac{1}{2}\right]$, and $r_i \in [0, s_i]$; $i, j = 1, 2;\ i \neq j$.

A2 $\frac{\partial^2 \Gamma_i}{\partial g_i^2} < 0$ and $\left|\frac{\partial^2 \Gamma_i}{\partial g_i^2}\right| > \left|\frac{\partial^2 \Gamma_i}{\partial g_i \partial g_j}\right| > \left|\frac{\partial^2 \Gamma_i}{\partial g_j^2}\right|$ for all $\alpha_i \in \{0, 1\}$, $s_i \in \left[0, \frac{1}{2}\right]$, and $r_i \in [0, s_i]$; $i, j = 1, 2;\ i \neq j$. (Second-order and stability conditions).

Note that A2 implies that $H = \frac{\partial^2 \Gamma_i}{\partial g_i^2} \frac{\partial^2 \Gamma_j}{\partial g_j^2} - \frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} \frac{\partial^2 \Gamma_j}{\partial g_i \partial g_j} > 0,\ i, j = 1, 2$ and $i \neq j$. Moreover, when $s_i = s_j = 0$, then $\Gamma_i(g_i, g_j) = \pi_i(g_i, g_j)$; hence, our assumption holds for the underlying game without cross-ownership, too. Note also that our assumption implies that the iso-profit curves are continuously differentiable and strictly concave (strictly convex) if $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} < (>) 0$.

We do not make any a priori assumption regarding the type of goods produced by firms and the nature of firms' product market strategies. That is, goods may be substitutes ($\frac{\partial \pi_i}{\partial g_j} < 0$) or complements ($\frac{\partial \pi_i}{\partial g_j} > 0$), and firms' product market strategies may be strategic substitutes ($\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} < 0$) or strategic complements ($\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} > 0$). From (2) it is evident that if $\frac{\partial^2 \pi_i}{\partial g_j \partial g_i} < (>) 0\ \forall\ i, j = 1, 2;\ i \neq j$, then $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} < (>) 0$ also holds true for all $i, j = 1, 2;\ i \neq j$. That is, if the underlying game without cross-ownership is of strategic substitutes (strategic complements), then the game with cross-ownership is also of strategic substitutes (strategic complements). Yet, the reverse is not necessarily true, unless $sign\ \left(\frac{\partial^2 \pi_1}{\partial g_2 \partial g_1}\right) = sign\ \left(\frac{\partial^2 \pi_2}{\partial g_1 \partial g_2}\right)$. Notice that our assumption allows for the mixed case in which from firm i's perspective strategies are strategic substitutes, while from firm j's perspective, they are strategic complements.

We consider the following three-stage game with *observable actions*.

Stage 1: Firm 1 and Firm 2 make simultaneous take-it-or-leave-it offers, (r_1, F_1) and (r_2, F_2), to outside investors, I_1 and I_2, respectively.

Stage 2: Each outside investor accepts or rejects its own offer.

Stage 3: Firms engage in product market competition.

The mode of product market competition is exogenously given and is common knowledge. To solve the game, we employ subgame perfectness. In section 4, we extend our analysis to a sequential move game in which in stage 3 and stage 4, the leader and the follower chooses its strategy, respectively.

3. Equilibrium analysis

In the last stage of the game, firm i's manager solves $\max_{g_i} \Gamma_i(g_i, g_j, r_i, r_j)$, where $\Gamma_i(\cdot)$ is given by (2). The first-order conditions are,

$$\frac{\partial \Gamma_i}{\partial g_i} = \frac{1}{1-(s_i-\alpha_i\, r_i)(s_j-\alpha_j\, r_j)}\left[\frac{\partial \pi_i}{\partial g_i} + (s_i-\alpha_i\, r_i)\frac{\partial \pi_j}{\partial g_i}\right] = 0. \qquad (3)$$

From (3), it is evident that if $s_i - \alpha_i\, r_i > 0$, $\frac{\partial \pi_j}{\partial g_i} < (>) 0 \Leftrightarrow \frac{\partial \pi_i}{\partial g_i} > 0\ (< 0)$. That is, if goods are substitutes (complements), internalization of rival's operating profit due to cross-holdings induces firms to be less (more) aggressive in the product market compared to that in absence of cross- holdings. The reason is that, compared to the case of no cross-holdings, cross-holdings induced less (more) aggressive play by a firm increases the rival's profit and that overcompensates the respective loss in own profit when goods are substitutes (complements).

Suppose for the moment that $\alpha_i = 1$, $i = 1, 2$, i.e., each firm's divestment offer has been accepted. Let $g_i^*(r_i, r_j)$, $\pi_i^*(r_i, r_j) = \pi_i\left(g_i^*(r_i, r_j), g_j^*(r_i, r_j)\right)$, and $\Gamma_i^*(r_i, r_j)$ be the equilibrium strategy, operating, and accounting profits of firm i in the last stage of the game, which are obtained by solving firms' first-order conditions (3) and using (2). From the comparative static analysis with respect to r_i and r_j we obtain the following Lemma.

Lemma 1: *For all* $i, j = 1, 2,\ i \neq j$, *it holds that:*

(a) $\frac{\partial g_i^*}{\partial r_i} > 0\ (< 0)$, *if firms produce substitute (complement) goods.*

(b) $\frac{\partial g_i^*}{\partial r_j} > 0$, *if (i) goods are substitutes and firms' strategies are strategic complements, or (ii) goods are complements and firms' strategies are strategic substitutes; otherwise,* $\frac{\partial g_i^*}{\partial r_j} < 0$.

Proof: See Appendix.

Intuitively, a higher divestment of cross-holdings by a firm reduces that firm's incentives to internalize its rival's profit, which in turn induces that firm to behave more (less) aggressively in case goods are substitutes (complements). On the other hand, more aggressive play by a firm induces its rival firm to play less (more) aggressively, if strategies are strategic substitutes (strategic complements). Thus, if goods are substitutes (complements) and strategies are strategic substitutes (strategic complements), a higher divestment by a firm induces its rival firm to behave less aggressively. The opposite happens in case goods are complements (substitutes) and the firms' strategic variables are strategic substitutes (complements).

For instance, if firms compete in quantities, their strategies are (typically) strategic substitutes (complements) when the goods are substitutes (complements). In this case, Lemma 1(b) informs us that as the rival firm's divestment increases, the firm always behaves less aggressively. The opposite occurs if firms compete in prices, in which case firms' strategies are (typically) strategic complements (substitutes) when the goods are substitutes (complements). On the other hand, Lemma 1(a) tells us that, independently whether firms compete in quantities or prices, if a firm's divestment increases, the firm behaves more (less) aggressively when goods are substitutes (complements).

In stage 2, outside investor I_i's valuation of r_i fraction of passive shares in firm j is equal to $r_i \Gamma_j^*(r_i, r_j)$. Thus, investor I_i accepts the offer (r_i, F_i), if $r_i \Gamma_j^*(r_i, r_j) \geq F_i$; otherwise, it rejects the offer. In other words, investor I_i's maximum willingness to pay for firm i's divested equity is equal to $r_i\Gamma_j^*(r_i, r_j)$. In stage 1, it is optimal for firm i to set $F_i = r_i\Gamma_j^*(r_i, r_j)$ and in stage 2 investor I_i accepts the offer for all $r_i \geq 0$.

Now, by substituting $F_i = r_i\Gamma_j^*(r_i, r_j)$, $\alpha_i = 1$, and $g_i = g_i^*(r_i, r_j)$ in (1), we get:

$$\Gamma_i^*(r_i, r_j) = \pi_i\left(g_i^*(r_i, r_j), g_j^*(r_i, r_j)\right) + s_i\Gamma_j^*(r_i, r_j).$$

Solving the system of equations, we obtain:

$$\Gamma_i^*(r_i, r_j) = \frac{\pi_i\left(g_i^*(r_i, r_j), g_j^*(r_i, r_j)\right) + s_i\,\pi_j\left(g_i^*(r_i, r_j), g_j^*(r_i, r_j)\right)}{1 - s_i s_j} \tag{4}$$

Clearly, divestment levels, (r_i, r_j), affect firms' accounting profits only indirectly via their effects on the equilibrium levels of the strategic variables, $g_i^*((r_i, r_j)$ and $g_j^*((r_i, r_j)$, and there is no direct effect. Now, from (4) we get,

$$\frac{\partial\Gamma_i^*(r_i, r_j)}{\partial r_i} = \frac{1}{1 - s_i s_j}\left[\left(\frac{\partial\pi_i^*}{\partial g_i} + s_i\frac{\partial\pi_j^*}{\partial g_i}\right)\frac{\partial g_i^*}{\partial r_i} + \left(\frac{\partial\pi_i^*}{\partial g_j} + s_i\frac{\partial\pi_j^*}{\partial g_j}\right)\frac{\partial g_j^*}{\partial r_i}\right]$$

Then, from (3) we have that $\frac{\partial\pi_i^*}{\partial g_i} + s_i\frac{\partial\pi_j^*}{\partial g_i} = r_i\frac{\partial\pi_j^*}{\partial g_i}$ and $\frac{\partial\pi_j^*}{\partial g_j} = -(s_j - r_j)\frac{\partial\pi_i^*}{\partial g_j}$. Hence, we get,

$$\frac{\partial\Gamma_i^*(r_i, r_j)}{\partial r_i} = \underbrace{\frac{r_i}{1 - s_i s_j}\frac{\partial\pi_j^*}{\partial g_i}\frac{\partial g_i^*}{\partial r_i}}_{\substack{\text{Effect via investor's profit} \\ (-)}} + \underbrace{\frac{1 - s_i(s_j - r_j)}{1 - s_i s_j}\frac{\partial\pi_i^*}{\partial g_j}\frac{\partial g_j^*}{\partial r_i}}_{\substack{\text{Effect due to rival's strategic response} \\ (-),\ \text{if strategic complements} \\ (+),\ \text{if strategic substitutes}}} \tag{5}$$

For any given $r_j(\geq 0)$, divestment of cross-holdings by firm i affects its accounting profit through two channels. First, divestment by firm i induces it to change the level of its own strategic variable, $g_i^*(.)$, which in turn affects firm j's operating profit, $\pi_j^*(.)$, and thus, affects the amount of fixed fee that can be charged by firm i to its investor I_i. By Lemma 1(a), if goods are substitutes (complements), i.e., if $\frac{\partial\pi_j^*}{\partial g_i} < 0$ (> 0), a higher divestment by firm i induces it to be more (less) aggressive in the product market, $\frac{\partial g_i^*}{\partial r_i} > 0$ (< 0). This leads to a lower accounting profit of the rival and, thus, a lower profit for the investor I_i, implying a lower fixed fee per unit of divested shares and a lower accounting profit of firm i. Therefore, regardless of whether goods are substitutes or complements, the effect of divestment via investor's profit is negative, which is captured by the first term of the right-hand-side of (5).

Second, divestment by firm i affects its operating profit, $\pi_i^*(.)$, via firm j's strategic variable, $g_j^*(.)$, due to strategic repositioning of firm j in response to firm i's divestment, which is captured by the second term on the right-hand-side of (5). Note that the $Sign$ of the latter is equal to $Sign\left(\frac{\partial \pi_i^*}{\partial g_j}\frac{\partial g_j^*}{\partial r_i}\right)$. Now, $\frac{\partial \pi_i^*}{\partial g_j} < (>) 0$ if goods are substitutes (complements), while $Sign\left(\frac{\partial g_j^*}{\partial r_i}\right)$ depends on both (a) the type of goods produced by firms (substitutes or complements) and (b) the nature of firms' strategic variables (strategic substitutes or strategic complements) — see Lemma 1(b). First, suppose that goods are substitutes (complements) and strategies are strategic substitutes, $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} < 0$. Then a higher divestment by firm i makes it more (less) aggressive in the product market, which induces its rival firm j to be less (more) aggressive. Therefore, if firms' strategies are strategic substitutes, the second effect is positive. Next, consider that goods are substitutes (complements), but strategies are strategic complements, $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} > 0$. Then, a higher divestment by firm i makes it more (less) aggressive, and that, in turn, induces its rival firm to be more (less) aggressive. Thus, if firms' strategies are strategic complements, the second effect of divestment is negative.

Therefore, it follows that, when firms' strategies are strategic complements, a higher divestment of cross-holdings by a firm always reduces its accounting profit and, thus, its manager has no incentive to divest any fraction of its passive shares of the rival firm. On the other hand, when firms' strategies are strategic substitutes, (5) implies that $\left.\frac{\partial \Gamma_i^*(r_i,r_j)}{\partial r_i}\right|_{r_i=0} = \left.\frac{1-s_i(s_j-r_j)}{1-s_i s_j}\frac{\partial \pi_i^*}{\partial g_j}\,\frac{\partial g_j^*}{\partial r_i}\right|_{r_i=0} > 0$, irrespective of whether goods are substitutes or complements. Therefore, in case strategies are strategic substitutes, firms always have incentives to divest some fraction of their respective cross-holdings.

Proposition 1: *The following hold regardless of whether goods are substitutes or complements:*

(a) If strategies are strategic substitutes, each firm has unilateral incentive to divest some fraction of its cross-holdings in the other firm to an outside investor.

(b) If strategies are strategic complements, it is optimal for a firm not to divest any fraction of its cross-holdings in the other firm to an outside investor.

Proof: The proof is immediate from Lemma 1 and equation (5).

Example 1: This example illustrates the above result considering a differentiated goods duopoly with linear demand functions and identical constant returns to scale production technologies.

Let firm i's demand be $p_i = a - q_i - \gamma q_j, i, j = 1, 2$ and $i \neq j$, where $a > 0$ is a measure of the market size, q_i is firms i's quantity, and $\gamma \in (-1, 1)$ is the product differentiation parameter. It is evident that goods are substitutes (complements) if $0 < \gamma < 1$ ($-1 < \gamma < 0$). Let the cost function of firm i be $C_i = cq_i$, $0 \leq c < a$.

We first consider that in stage 3, firms compete by setting simultaneously their quantities (Cournot game). Next, we consider that there is simultaneous move price competition (Bertrand game).

Cournot Competition: In this case, firm i's strategic variable is $g_i = q_i$, and $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} = \frac{\partial^2 \Gamma_i}{\partial q_j \partial q_i} = -\gamma(1 + s_2 - r_2)\begin{cases} < 0, \ if \ \ 0 < \gamma < 1 \\ > 0, \ if - 1 < \gamma < 0 \end{cases}$. It follows that if $0 < \gamma < 1$, i.e., if goods are substitutes, firms' strategies are strategic substitutes. Otherwise, if $-1 < \gamma < 0$, i.e., if goods are complements, firms' strategies are strategic complements. Now, solving the stage 3 equilibrium outputs and substituting those in the right-hand-side (RHS) of equation (4), we obtain the expression for $\Gamma_i^*(r_i, r_j)$ under Cournot competition. It can be checked that,

$$\left.\frac{\partial \Gamma_i^*(r_i, r_j)}{\partial r_i}\right|_{r_i=0} = \frac{(a-c)^2\gamma^3(2-\gamma-\gamma s_i)(1-s_i(s_j-r_j))(1+s_j-r_j)(2-\gamma+\gamma r_j-\gamma s_j)}{(1-s_i s_j)(4-\gamma^2+\gamma^2(r_j(1+s_i)-s_j-s_i(1+s_j))^3}\begin{cases} > 0, if \ 0 < \gamma < 1 \\ < 0, if \ \ -1 < \gamma < 0 \end{cases}.$$

Therefore, under Cournot competition, the following are true: (a) If $0 < \gamma < 1$, i.e., if goods are substitutes, firms' strategies are strategic substitutes, and it is optimal for a firm to divest some of its cross-holdings regardless of whether its rival divests or not. (b) If $-1 < \gamma < 0$, i.e., if goods are complements, firms' strategies are strategic complements, and no firm has an incentive to divest its cross-holdings to an outside investor.

Bertrand competition: Under price competition, firm i's strategic variable can be defined as $g_i = -p_i$; then we get $\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i} = \frac{\partial^2 \Gamma_i}{\partial p_j \partial p_i} = \frac{\gamma(1+s_2-r_2)}{1-\gamma^2}\begin{cases} > 0, \ if \ \ 0 < \gamma < 1 \\ < 0, \ if - 1 < \gamma < 0 \end{cases}$. It follows that if $0 < \gamma < 1$, i.e., if goods are substitutes, firms' strategies are strategic complements. Otherwise, if $-1 < \gamma < 0$, i.e., if goods are complements, firms' strategies are strategic substitutes. These are exactly opposites to those under Cournot competition. Solving the stage 3 equilibrium prices and substituting those in the RHS of equation (4), we obtain the expression for $\Gamma_i^*(r_i, r_j)$ under Bertrand competition. Then we get,

$$\left.\frac{\partial\Gamma_i^*(r_i,r_j)}{\partial r_i}\right|_{r_i=0} =$$

$$. \qquad -\frac{(a-c)^2(1-\gamma)\gamma^3(2+\gamma+\gamma s_i)(1-s_i(s_j-r_j))(1+s_j-r_j)(2+\gamma(1+s_j-r_j))}{(1+\gamma)(1-s_is_j)(4-\gamma^2-\gamma^2(s_j-r_j+s_i(1+s_j-r_j)))^3}\begin{cases}<0, if\ 0<\gamma<1\\ >0, if\ -1<\gamma<0\end{cases}.$$

This implies that the following are true under Bertrand competition: (a) If $0 < \gamma < 1$, i.e., if goods are substitutes, firms' strategies are strategic complements, and no firm has an incentive to divest its cross-holdings to an outside investor, (b) If $-1 < \gamma < 0$, i.e., if goods are complements, firms' strategies are strategic substitutes, and it is optimal for a firm to divest some of its cross-holdings regardless of whether its rival divests or not.

Therefore, our findings under Cournot and Bertrand competition taken together imply that $\frac{\partial^2\Gamma_i}{\partial g_j \partial g_i} > (< 0) \Rightarrow \left.\frac{\partial\Gamma_i^*(r_i,r_j)}{\partial r_i}\right|_{r_i=0} < (> 0)$, regardless of whether $\gamma \in (0,1)$ or $\gamma \in (-1,0)$, as in Proposition 1.

Since Proposition 1(a) holds true for any arbitrary $s_i \in \left(0, \frac{1}{2}\right], i = 1,2$, we can state the following.

Proposition 2: *Suppose that firms' strategies are strategic substitutes. Then, regardless of whether goods are substitutes or complements, it is optimal for firms to divest their respective cross-holdings fully to outside investors.*

Proof: See Appendix.

Clearly, stability of cross-ownership depends on the nature of firms' strategies – strategic substitutes or strategic complements -, and not on the type of goods – substitutes or complements. We, thus, demonstrate that the result of Stenbacka and Moer (2021) holds true not only for substitute goods, but also in case goods are complements. However, when firms' strategies are strategic complements, Stenbacka and Moer (2021)'s result is completely reversed.

4. Alternative Divestment Mechanisms

We have considered above that firms sell their respective passive shares in rival firms to different outside investors. In this section we consider implications of two alternative mechanisms of divestments, namely competitive pricing, and private placement via common intermediary.

Competitive bidding

Suppose that firms choose to divest their passive shares of their rivals through some competitive bidding process, e.g., a Dutch auction. In the latter, potential investors submit their bids for the number of shares and the price they are willing to pay for the stock. The highest price at which all the shares can be sold is determined as the offering price, and all successful bidders pay that price.

Firm i's maximum possible return from divesting r_i passive shares of its rival is equal to $F_i = r_i \Gamma_j^*$. The average price of firm i's shares determined through the bidding process is $\mu_i \Gamma_j^*$, $0 < \mu_i \leq 1$. Then, firm i's receipts from divestment are equal to $r_i \mu_i \Gamma_j^*$ and its post divestment accounting profit is:

$$\Gamma_i^*(r_i, r_j, \mu_i, \mu_j) = \pi_i\left(g_i^*(r_i, r_j), g_j^*(r_i, r_j)\right) + (s_i - r_i(1-\mu_i))\Gamma_j^*(r_i, r_j, \mu_i, \mu_j).$$

Hence,

$$\Gamma_i^*(r_i, r_j, \mu_i, \mu_j) = \frac{\pi_i\left(g_i^*(r_i,r_j), g_j^*(r_i,r_j)\right) + (s_i - r_i(1-\mu_i))\pi_j\left(g_i^*(r_i,r_j), g_j^*(r_i,r_j)\right)}{1 - (s_i - r_i(1-\mu_i))\left(s_j - r_j(1-\mu_j)\right)}.$$

Comparing this expression for $\Gamma_i^*(.)$ with that in (4), it follows that if μ_i is close to 1, Propositions 1 and 2 will hold true. As a competitive bidding with many potential investors is expected to lead to an average price close to Γ_j^*, i.e., μ_i close to 1, we can safely infer that our main findings hold under this alternative divestment mechanism too.

Private placement via a common intermediary

Let now firms divest their passive cross-holdings of their rivals via a common intermediary through private placement. Divestment through private placement is possible only if total payoff of all agents involved (i.e., payoffs of buyers of divested shares plus payoffs of firms) increases due to divestment.

Suppose that $\alpha_i = \alpha_j = 1$ (i.e., the intermediary agrees to help divesting) and $F_i = r_i \Gamma_j^*$ (i.e., divested shares are sold at the highest possible price). Then $\Gamma_i^*(r_i, r_j) = \frac{\pi_i(.) + s_i \pi_j(.)}{1 - s_i s_j}$ and the sum of accounting profits of firms (i.e., total payoff of all agents involved) is given by,

$$Z^*(r_i, r_j) = \Gamma_i^*(r_i, r_j) + \Gamma_j^*(r_i, r_j)$$

$$= \frac{1}{1 - s_i s_j} [(1 + s_j)\pi_i \left(g_i^*(r_i, r_j), g_j^*(r_i, r_j) \right) + (1 + s_i)\pi_j \left(g_i^*(r_i, r_j), g_j^*(r_i, r_j) \right)]$$

Then, $\frac{\partial Z^*}{\partial r_i} = \frac{1}{1 - s_i s_j} \left[(1 + s_j) \left(\frac{\partial \pi_i}{\partial g_i} \frac{\partial g_i^*}{\partial r_i} + \frac{\partial \pi_i}{\partial g_j} \frac{\partial g_j^*}{\partial r_i} \right) + (1 + s_i) \left(\frac{\partial \pi_j}{\partial g_i} \frac{\partial g_i^*}{\partial r_i} + \frac{\partial \pi_j}{\partial g_j} \frac{\partial g_j^*}{\partial r_i} \right) \right]$.

From the first-order conditions of firms' maximization problems in Stage 3 of the game, we have $\frac{\partial \pi_i}{\partial g_i} = -(s_i - r_i) \frac{\partial \pi_j}{\partial g_i} = 0$. Thus,

$$\frac{\partial Z^*}{\partial r_i} = \frac{1}{1 - s_i s_j} \left[(1 + s_j) \left(-(s_i - r_i) \frac{\partial \pi_j}{\partial g_i} \frac{\partial g_i^*}{\partial r_i} + \frac{\partial \pi_i}{\partial g_j} \frac{\partial g_j^*}{\partial r_i} \right) + (1 + s_i) \left(\frac{\partial \pi_j}{\partial g_i} \frac{\partial g_i^*}{\partial r_i} - (s_j - r_j) \frac{\partial \pi_i}{\partial g_j} \frac{\partial g_j^*}{\partial r_i} \right) \right]$$

$$= \frac{1}{1 - s_i s_j} \left[\frac{\partial \pi_j}{\partial g_i} \frac{\partial g_i^*}{\partial r_i} \left((1 + s_i) - (1 + s_j)(s_i - r_i) \right) + \frac{\partial \pi_i}{\partial g_j} \frac{\partial g_j^*}{\partial r_i} \left((1 + s_j) - (1 + s_i)(s_j - r_j) \right) \right] \quad (8)$$

Using Lemma 1 and recalling that if goods are substitutes (complements) $\frac{\partial \pi_j}{\partial g_i} < 0 \ (> 0)$, we get the following Proposition.

Proposition 3: *If divestment occurs via a common intermediary,*

(a) when firms' strategies are strategic complements, there are no divestment incentives of cross-holdings, independently whether goods are substitutes or complements.

(b) when firms' strategies are strategic substitutes and firms are symmetric in all aspects, firms have no incentive to divest, independently whether goods are substitutes or complements. Otherwise, they may or may not have incentives to divest some fraction of their cross-holdings depending on the specific market features.

Proof: See Appendix.

Proposition 3 partially only confirms our main findings for the case that private placement takes place via a common intermediary. In particular, when strategies are strategic complements, once again there are no divestment incentives, independently whether goods are substitutes or complements. In contrast, when strategies are strategic substitutes, it is not always true that a firm has always incentives to divest its cross-holdings of the rival. Interestingly, when firms are ex-ante symmetric in all aspects and their goods are substitutes, they have no incentives for divestment. In all other cases, they may or may not have such incentives.

Example 2: This example illustrates divestment of passive shares in case of strategic substitutability. Suppose that there are two firms, 1 and 2, producing homogeneous goods at marginal costs c_1 and c_2, where $0 \leq c_1 < c_2$, and are engaged in Cournot competition in the product market. The inverse market demand function is given by $p = a - q_1 - q_2$; $a > 2c_2 - c_1$. Firm 1 owns $s_1 \in (0, \frac{1}{2}]$ passive shares in firm 2. Accounting profits of firm 1 and firm 2 are, respectively, $\Gamma_1 = \pi_1 + s_1\Gamma_2$ and $\Gamma_2 = \pi_2$, which implies that $\Gamma_1 = \pi_1 + s_1\pi_2$ and $\Gamma_2 = \pi_2$. Then, for any given s_1, the equilibrium outputs and accounting profits are, respectively,

$$q_1(s_1) = \frac{(1-s_1)a - 2c_1 + (1+s_1)c_2}{3-s_1}, \qquad \Gamma_1 = \frac{(a-2c_1+c_2)^2 - (5a-c_1-4c_2)(c_2-c_1)s_1 + (a-c_2)(c_2-c_1)s_1^2}{(3-s_1)^2},$$

$$q_2(s_1) = \frac{a+c_1-2c_2}{3-s_1}, \qquad \Gamma_2 = \frac{(a+c_1-2c_2)^2}{(3-s_1)^2}.$$

Now, $\frac{\partial \Gamma_1}{\partial s_1} = \frac{(a+c_1-2c_2)[2a-7c_2+5c_1+s_1(c_2-c_1)]}{(3-s_1)^3} < 0$, if $2c_2 - c_1 < a < \frac{1}{2}[7c_2 - 5c_1 - s_1(c_2 - c_1)]$. This implies that, if the market size is less than a critical level, it is optimal for the more efficient firm not to own any share of its rival firm. Therefore, if firms are sufficiently heterogeneous in terms of their marginal costs, market size is small and cross-ownership structure is exogenously given, the more efficient firm will have an incentive to fully divest its passive cross-holdings, even under divestment through private placement via a common intermediary. In contrast, it can be checked that, if the efficiency gap between the firms is less than a critical level such that $\left|\frac{\partial \pi_2}{\partial q_1}\right| = q_2(s_1) \geq \left|\frac{\partial \pi_1}{\partial q_2}\right| = q_1(s_1)$, we will have $\frac{\partial \Gamma_1}{\partial s_1} > 0$. In the latter case, divestment of passive shares will not occur through private placement via common intermediary.

5. Sequential move product market competition

We next consider that firms' strategic interaction is sequential. Let firm 1 (the leader) choose the level of g_1 in stage 3 and then firm 2 (the follower) choose the level of g_2 in stage 4. The first two stages of the game, as well as all other specifications, are as in the main model. For tractability of the analysis, we make the following assumption:

Assumption:

A3. $sign\left(\frac{\partial^2 \pi_1}{\partial g_2 \partial g_1}\right) = sign\left(\frac{\partial^2 \pi_2}{\partial g_1 \partial g_2}\right)$

The above assumption holds true even when firms face different demand functions and/or their cost functions are different, except in some special cases.[6] It implies that, if the underlying game with cross-ownership is of strategic substitutes (strategic complements), then the game without cross-ownership is also of strategic substitutes (strategic complements), and vice-versa.

The follower's problem in stage 4 is to $\max_{g_2} \Gamma_2(g_1, g_2, r_1, r_2)$, where $\Gamma_2(\cdot)$ is given by (2) when $i = 2$. The first order condition of this problem is given by (3) for $i = 2$. Letting $\alpha_2 = 1$, the solution to the follower's problem is $g_{2,RF}(g_1, r_1, r_2)$, which is the reaction function of firm 2. However, it is evident that $\frac{\partial g_{2,RF}}{\partial r_1} = 0$, i.e., divestment by the leader does not have any direct impact on the follower's reaction function; thus, we write $g_{2,RF}(g_1, r_2)$. Further, it can be checked that (i) $\frac{\partial g_{2,RF}}{\partial g_1} < (>) 0$, if $\frac{\partial^2 \Gamma_2}{\partial g_2 \partial g_1} < (>) 0$, i.e., the follower's reaction function is negatively (positively) sloped if strategies are strategic substitutes (strategic complements); and (ii) $\frac{\partial g_{2,RF}}{\partial r_2} > (<) 0$, if $\frac{\partial \pi_2}{\partial g_1} < (>) 0$, i.e., divestment of the follower's share in the leader firm induces the follower, for any given g_1, to be more (less) aggressive in the case of substitute (complement) goods. (See Appendix for details.)

Now, assuming that $\alpha_1 = 1$, the first order condition of the leader's problem in stage 3, $\max_{g_1} \Gamma_1(g_1, g_2, r_1)$, where $g_2 = g_{2,RF}(g_1, r_2)$, implies that $\frac{\partial \Gamma_1}{\partial g_1} + \frac{\partial \Gamma_1}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1} = 0$. Let $g_1^L(r_1, r_2)$ and

[6] See Bulow et al. (1985) for a discussion on this issue.

$g_2^F(r_1, r_2) \equiv g_{2,RF}(g_1^L(r_1, r_2), r_2)$ be the equilibrium strategy of the leader and the follower, respectively, when $\alpha_1 = \alpha_2 = 1$. Further, let $\pi_1^L(r_1, r_2) = \pi_1\big(g_1^L(r_1, r_2), g_2^F(r_1, r_2)\big)$ and $\Gamma_1^L(r_1, r_2)$ be the leader's operating and accounting profits, with $\pi_2^F(r_1, r_2) = \pi_2\big(g_1^L(r_1, r_2), g_2^F(r_1, r_2)\big)$, and $\Gamma_2^F(r_1, r_2)$ those of the follower.

From the comparative static analysis of the last two stages equilibrium outcomes corresponding to $\alpha_1 = \alpha_2 = 1$, we get the following Lemma.

Lemma 2: *It holds that:*

(a) $\frac{\partial g_1^L}{\partial r_1} > 0\ (< 0)$ *and* $\frac{\partial g_2^F}{\partial r_2} > (<) 0$, *if goods are substitutes (complements).*

(b) $\frac{\partial g_1^L}{\partial r_2} > 0$ *and* $\frac{\partial g_2^F}{\partial r_1} > 0$ *if (i) goods are substitutes and firms' strategies are strategic complements, or (ii) goods are complements and firms' strategies are strategic substitutes; otherwise,* $\frac{\partial g_1^L}{\partial r_2} < 0$ *and* $\frac{\partial g_2^F}{\partial r_1} < 0$.

Proof: See Appendix

From Lemma 1 and 2, it is evident that the impact of divestments on firms' equilibrium strategies in the case of sequential move competition are similar in nature to those in the case of simultaneous move competition. Intuitions are also like those in the case of simultaneous moves.

In stage 1, like in the case of simultaneous moves, it is optimal for the leader (follower) to set $F_1^L = r_1\Gamma_2^F(r_1, r_2)$ ($F_2^F = r_2\Gamma_1^L(r_1, r_2)$). Then in stage 2, each investor accepts its offer for all $r_i \geq 0,\ i = 1, 2$. Thus, firms' accounting profits can be written as:

Leader: $$\Gamma_1^L(r_1, r_2) = \frac{\pi_1^L\big(g_1^L(r_1, r_2), g_2^F(r_1, r_2)\big) + s_1\pi_2^F(\, g_1^L(r_2, r_1), g_2^F(r_1, r_2))}{1 - s_1 s_2}$$

Follower: $$\Gamma_2^F(r_2, r_1) = \frac{\pi_2^F(\, g_1^L(r_2, r_1), g_2^F(r_1, r_2)) + s_2\pi_1^L\big(g_1^L(r_1, r_2), g_2^F(r_1, r_2)\big)}{1 - s_1 s_2}$$

We examine next whether there are incentives of the leader and/or the follower to divest its passive shares in its rival. First, consider the leader's divestment incentives. From the above expression, the slope of the leader's accounting profit with respect to r_1 can be decomposed as follows: (See proof of Proposition 4 in the Appendix for details.)

$$\frac{d\Gamma_1^L(\cdot)}{dr_1} = \frac{1}{1-s_1s_2}\left[\frac{d\pi_1^L(.)}{dr_1} + s_1\frac{d\pi_2^F(.)}{dr_1}\right]$$

$$= \underbrace{\left[\frac{r_1}{1-s_1s_2}\frac{\partial\pi_2^F}{\partial g_1}\right]\frac{\partial g_1^L}{\partial r_1}}_{(-)} + \underbrace{\left[-\frac{1-(s_1-r_1)(s_2-r_2)}{1-s_1s_2}\frac{\partial\pi_1^L}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right]\frac{\partial g_1^L}{\partial r_1}}_{\substack{(-),\ \text{if strategic substitutes}\\(+),\ \text{if strategic complements}}} + \underbrace{\left[\frac{1-s_1(s_2-r_2)}{1-s_1s_2}\frac{\partial\pi_1^L}{\partial g_2}\right]\frac{\partial g_2^F}{\partial r_1}}_{\substack{(+),\ \text{if strategic substitutes}\\(-),\ \text{if strategic complements}}} \tag{7}$$

$$= \underbrace{\frac{1}{1-s_1s_2}\left[r_1\frac{\partial\pi_2^F}{\partial g_1}\frac{\partial g_1^L}{\partial r_1}\right]}_{\substack{\text{Effect via investor's profit}\\(-)}} + \underbrace{\frac{1}{1-s_1s_2}\left[-r_1(s_2-r_2)\frac{\partial\pi_1^L}{\partial g_2}\right]\frac{\partial g_2^F}{\partial r_1}}_{\substack{\text{Effect due to rival's strategic response}\\(-),\ \text{if strategic substitutes}\\(+),\ \text{if strategic complements}}} \tag{8}$$

Equation (8) decomposes the effect of divestment of cross-holdings by the leader on its accounting profit, $\frac{\partial\Gamma_1^L(\cdot)}{\partial r_1}$, into two parts: The first term captures the *effect via investor's profit*, which is always negative as in the case of simultaneous move competition. The second term captures the *effect due to follower's strategic response* to divestment by the leader. Divestment by the leader firm 1 induces it to re-optimize its strategy, g_1, and that, in turn, induces the follower firm 2 to respond strategically by changing the level of its strategic variable, g_2, which affects the leader's profit, π_1^L. The leader takes this into account while choosing its optimal strategy, unlike in the case of simultaneous move competition.

Therefore, the *effect due to follower's strategic response* has two components: First, the effect due to internalization of the follower's strategic response by the leader while choosing its strategy - the second term of equation (7) -, which is absent under simultaneous move competition; and second, the effect due to the follower's strategic response even in absence of internalization by the leader - the third term of equation (7). The latter is analogous to the second term of equation (5) under simultaneous move competition. The first component of the *effect due to follower's strategic response* is negative (positive) in the case of strategic substitutes (strategic complements) and is opposite in sign to that of the second term. Yet, it is larger in magnitude than the second term. Thus, under sequential moves the *effect due to follower's strategic response* is negative (positive) in the case of strategic substitutes (strategic complements).

Simplifying the RHS of equation (8) further, we get,

$$\frac{\partial\Gamma_1^L(\cdot)}{\partial r_1} = \frac{1}{1-s_1s_2}\ r_1\ \left(\frac{\partial g_1^L}{\partial r_1}\right)^2 \frac{d}{dg_1}\left(\frac{d\,\Gamma_1(g_1, g_{2,RF}(g_1,r_2),\ r_1)}{dg_1}\right) < 0 \tag{9}$$

By the second order conditions of the leader's maximization problem, the latter is negative for all $r_1 \in (0, s_1]$. Clearly, it is optimal for the leader to choose $r_1 = 0$ in stage 1, regardless of the nature of goods produced by firms – substitutes or complements – and the nature of firms' strategic variables – strategic substitutes or strategic complements.

Turning to the follower's problem of divestment in stage 1, we find that $Sign\left(\frac{\partial \Gamma_2^F(\cdot)}{\partial r_2}\Big|_{r_2=0}\right) = -Sign\left(\frac{\partial g_1^L}{\partial r_1}\frac{\partial g_1^L}{\partial r_2}\right)$. Thus, from Lemma 2, it follows that $\frac{\partial \Gamma_2^F(\cdot)}{\partial r_2}\Big|_{r_2=0} \begin{cases} > 0, \text{if firms' strategies are strategic substitutes} \\ < 0, \text{if firms' strategies are strategic complements} \end{cases}$. That is, the follower has no incentives to divest a fraction of its cross-holdings in the leader if firms' strategies are strategic complements. In contrast, if firms' strategies are strategic substitutes, the follower has always such incentives. This result is in line with the case of simultaneous move competition since the follower's stage 4 decision problem is similar in nature as that under simultaneous moves.

Proposition 4: *The following hold in the sequential move game:*

(a) The leader has no incentives to divest its passive cross-holdings in the follower to an outside investor, regardless of whether goods are substitutes or complements and firms' strategies are strategic substitutes or strategic complements.

(b) The follower has incentives to divest some fraction of its passive shares in the leader if firms' strategies are strategic substitutes. In contrast, if firms' strategies are strategic complements, the follower has no incentives to divest its cross-holdings, regardless of whether goods are substitutes or complements.

Proof: See Appendix.

It is always optimal for the leader not to divest any fraction of its passive shares in the follower to an outside investor. The same is true for the follower unless firms' strategies are strategic substitutes. In the latter case, for any given cross-ownership structure, the follower has incentives to divest some fraction of its cross-holdings. This implies that if strategies are strategic substitutes, it is optimal for the follower to fully divest its cross-holdings. Clearly, the present analysis encompasses the analysis of Stenbacka and Moer (2021) as a special case and demonstrates that strategic substitutability is not a sufficient condition for a firm to have incentives to divest some fraction of its passive shares in its rival.

Example 4: Let the demand functions faced by firms and their cost functions are as in Example 1. Further, consider that firm 1 is the leader and firm 2 is the follower. Then under sequential move quantity competition, for any given $s_i \in [0, \frac{1}{2}]$ and $r_i \in [0, s_i]$, firm 2's quantity reaction function in stage 4 and firm 1's equilibrium quantity in stage 3 are, respectively, $q_2 = \frac{1}{2}[a - c - \gamma q_1(1 + s_2 - r_2)]$ and $q_1 = \frac{(a-c)[2-\gamma(1+s_1-r_1)]}{4-\gamma^2[2+(s_1-r_1)(1-s_2+r_2)](1+s_2-r_2)}$. From these we get,

$$\frac{\partial \Gamma_1^L(\cdot)}{\partial r_1} = -\frac{(a-c)^2\gamma^2 r_1\left[4 - \gamma(2+\gamma) - \gamma(s_2 - r_2)\left(2\gamma - (2-\gamma)(s_2 - r_2)\right)\right]^2}{2(1 - s_1 s_2)\left[4 - \gamma^2\left(2 + (s_1 - r_1)(1 - s_2 + r_2)\right)(1 + s_2 - r_2)\right]^3} < 0,$$

for all $\gamma \in (-1, 0) \cup (1, 0), s_i \in \left[0, \frac{1}{2}\right]$ and $r_i \in [0, s_i]$, implying that the leader has no incentive to divest its cross-holdings in its rival (the follower). On the other hand,

$$\left.\frac{\partial \Gamma_2^F(\cdot)}{\partial r_2}\right|_{r_2=0} = \frac{(a-c)^2\gamma^3[2-\gamma(1+s_1-r_1)][1-(s_1-r_1)s_2]^2[4-\gamma(2+\gamma)+\gamma(2-\gamma)s_2^2-2\gamma^2 s_2]}{(1-s_1 s_2)[4-2\gamma^2(1+s_2)-\gamma^2(s_1-r_1)(1-s_2^2)]^3}\begin{cases} > 0, if\ 0 < \gamma < 1 \\ < 0, if -1 < \gamma < 0\text{'} \end{cases}$$

which implies that the follower has an incentive to divest its stake in its rival (the leader), if $0 < \gamma < 1$, i.e., if goods are substitutes and firms' strategies are strategic substitutes; otherwise, not.

Next, consider that there is sequential move price competition in the product market. In this case, for any given $s_i \in [0, \frac{1}{2}]$ and $r_i \in [0, s_i]$, the follower's price reaction function in stage 4 and the leader's equilibrium price in stage 3 are, respectively,

$p_2 = \frac{1}{2}\left[a(1-\gamma) + c\left(1 + \gamma(r_2 - s_2)\right) + \gamma p_1(1 + s_2 - r_2)\right]$, and

$$p_1 = \frac{a[2-\gamma^2(1+s_1-r_1)-\gamma(1-s_1+r_1)]+c[2+\gamma(1-s_1+r_1)+\gamma^2(-1+(s_1-r_1)(s_2-r_2)^2-2(s_2-r_2))]}{4-2\gamma^2-\gamma^2[(s_1-r_1)(1-(s_2-r_2)^2)+2(s_2-r_2)]}.$$

Thus,

$$\frac{\partial \Gamma_1^L(\cdot)}{\partial r_1} - \frac{(a-c)^2(1-\gamma)\gamma^2 r_1\left(-4-2\gamma+\gamma^2+\gamma(2+\gamma)r_2^2+2\gamma^2 s_2+\gamma(2+\gamma)s_2^2-2\gamma r_2(\gamma+(2+\gamma)s_2)\right)^2}{2(1+\gamma)(1-s_1 s_2)\left(4-2\gamma^2-\gamma^2 s_1+\gamma^2 r_2^2 s_1-2\gamma^2 s_2+\gamma^2 s_1 s_2^2-2\gamma^2 r_2(-1+s_1 s_2)-\gamma^2 r_1\left(-1+r_2^2-2r_2 s_2+s_2^2\right)\right)^3} < 0,$$

for all $\gamma \in (-1, 0) \cup (1, 0), s_i \in \left[0, \frac{1}{2}\right]$ *and* $r_i \in [0, s_i]$. In this case too, the leader has no incentive to divest its cross-holdings in its rival (the follower).

Turning to the follower's problem, we get,

$$\left.\frac{\partial \Gamma_2^F(\cdot)}{\partial r_2}\right|_{r_2=0} =$$

$$-\frac{(a-c)^2(1-\gamma)\gamma^3(2+\gamma(1+s_1-r_1))(1+(r_1-s_1)s_2)^2(4+(2-\gamma)\gamma-\gamma s_2(2\gamma+(2+\gamma)s_2))}{(1+\gamma)(1-s_1 s_2)(4-2\gamma^2+\gamma^2(r_1-s_1-2s_2+(-r_1+s_1)s_2^2))^3}\begin{cases} > 0, if\ -1 < \gamma < 0 \\ < 0, if\ 0 < \gamma < 1 \end{cases}.$$

This implies that the follower has an incentive to divest its stake in its rival (the leader) if $-1 < \gamma < 0$, i.e., if goods are complements and firms' strategies are strategic substitutes; otherwise, not.

6. Concluding Remarks

In a general duopolistic market context in which goods are either substitutes or complements and firms' strategies are either strategic substitutes or strategic complements, we have examined the firms' incentives to divest their passive shares in their rivals. We have considered alternative divestment mechanisms: private placement via independent intermediaries as well as via a common intermediary and competitive bidding.

We have shown that there are always divestment incentives under a private placement mechanism via independent intermediaries *but only if* firms' strategies are strategic substitutes in a simultaneous move product market game. This holds independently whether goods are substitutes or complements. A similar result is obtained under a competitive bidding mechanism. Yet, under a private placement mechanism via a common intermediary, firms do not have divestment incentives when their strategies are strategic substitutes *unless* there are substantial asymmetries in firms' cross holdings as well as in other market characteristics. Finally, only the follower has divestment incentives in a sequential move product market game when firms' strategies are strategic substitutes, independently whether goods are substitutes or complements.

In all other cases, no firm has divestment incentives, implying that we can safely analyze the market and societal implications of firms' arbitrary passive cross-holdings with no need to question the stability of the ownership structure.

Our analysis leads to several testable implications. First, divestment activities are more often observed in industries in which firms' strategies are strategic substitutes than those in which they are strategic complements. Second, in industries in which firms' strategies are strategic substitutes, divestment activities are less often observed when divestment occurs via a common intermediary than via independent intermediaries or competitive bidding. Third, in industries in which firms' strategies are strategic substitutes, leaders are less often engaged in divestment activities than followers.

There are two maintained assumptions in our analysis. First, investors buying firms' passive shares have no bargaining power and pay, thus, the highest amount that firms ask for selling those shares. And second, these investors have no impact on the firms' managers product market decisions ex-post, i.e., once they have bought the competing firms' shares. Relaxing these assumptions may lead to interesting new findings regarding the firms' incentives to divest their cross-holdings, an issue which is left for future research.

Appendix

Proof of Lemma 1: When $\alpha_i = \alpha_j = 1$, equation (3) reduces to,

$$\frac{\partial \Gamma_i}{\partial g_i} = \frac{1}{1-(s_i - r_i)(s_j - r_j)}\left[\frac{\partial \pi_i}{\partial g_i} + (s_i - r_i)\frac{\partial \pi_j}{\partial g_i}\right] = 0$$

Setting $\Omega_i = \pi_i + (s_i - r_i)\pi_j$, $i, j = 1,2, i \neq j$, we have

$$\frac{\partial \Omega_i}{\partial g_i} = \frac{\partial \pi_i}{\partial g_i} + (s_i - r_i)\frac{\partial \pi_j}{\partial g_i} = 0 \tag{A1}$$

By totally differentiating equation (A1) with respect to r_i, we get.

$$\begin{cases} \frac{\partial^2 \Omega_i}{\partial g_i^2}\frac{\partial g_i}{\partial r_i} + \frac{\partial^2 \Omega_i}{\partial g_j \partial g_i}\frac{\partial g_j}{\partial r_i} + \left(-\frac{\partial \pi_j}{\partial g_i}\right) = 0 \text{ and} \\ \frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}\frac{\partial g_i}{\partial r_i} + \frac{\partial^2 \Omega_j}{\partial g_j^2}\frac{\partial g_j}{\partial r_i} = 0 \end{cases} \tag{A2}$$

$$\Leftrightarrow \begin{bmatrix} \frac{\partial^2 \Omega_i}{\partial g_i^2} & \frac{\partial^2 \Omega_i}{\partial g_j \partial g_i} \\ \frac{\partial^2 \Omega_j}{\partial g_i \partial g_j} & \frac{\partial^2 \Omega_j}{\partial g_j^2} \end{bmatrix} \begin{bmatrix} \frac{\partial g_i}{\partial r_i} \\ \frac{\partial g_i}{\partial r_i} \end{bmatrix} = \begin{bmatrix} \frac{\partial \pi_j}{\partial g_i} \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} \frac{\partial g_i}{\partial r_i} = \frac{\frac{\partial \pi_j}{\partial g_i}\frac{\partial^2 \Omega_j}{\partial g_j^2}}{\widetilde{H}} \quad \text{and} \\ \frac{\partial g_j}{\partial r_i} = \frac{-\frac{\partial \pi_j}{\partial g_i}\frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}}{\widetilde{H}}, \end{cases}$$

where $\widetilde{H} = \frac{\partial^2 \Omega_i}{\partial g_i^2}\frac{\partial^2 \Omega_j}{\partial g_j^2} - \frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}\frac{\partial^2 \Omega_i}{\partial g_j \partial g_i} = \left(1-(s_i - r_i)(s_j - r_j)\right)^2 H > 0$ by Assumption A2.

Also, note that $\frac{\partial^2 \Omega_i}{\partial g_j \partial g_i} = \left(1-(s_i - r_i)(s_j - r_j)\right)\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i}$. Thus, $Sign\left(\frac{\partial^2 \Omega_i}{\partial g_j \partial g_i}\right) = Sign\left(\frac{\partial^2 \Gamma_i}{\partial g_j \partial g_i}\right)$.

Now, we can state the following.

(1) $Sign\left(\frac{\partial g_i}{\partial r_i}\right) = Sign\left(\frac{\partial \pi_j}{\partial g_i}\right)\begin{cases} > 0, \text{if } \frac{\partial \pi_j}{\partial g_i} < 0 \\ < 0, \text{if } \frac{\partial \pi_j}{\partial g_i} > 0 \end{cases}$, since $\widetilde{H} > 0$ and

$\frac{\partial^2 \Omega_j}{\partial g_j^2} = \left(1-(s_i - r_i)(s_j - r_j)\right)\frac{\partial^2 \Gamma_j}{\partial g_j^2} < 0$ by Assumption A2 .

(2) $\frac{\partial g_j}{\partial r_i} = \begin{cases} < 0, \text{if both } \frac{\partial \pi_j}{\partial g_i} \text{ and } \frac{\partial^2 \Gamma_j}{\partial g_j \partial g_i} \text{ are of the same sign} \\ > 0, \text{if } \frac{\partial \pi_j}{\partial g_i} \text{ and } \frac{\partial^2 \Gamma_j}{\partial g_j \partial g_i} \text{ are of different signs} \end{cases}$, since $\widetilde{H} > 0$ and

$Sign\left(\frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}\right) = Sign\left(\frac{\partial^2 \Gamma_j}{\partial g_j \partial g_i}\right)$. ■

Proof of Proposition 2:

Let $s_i^* \in (0,\frac{1}{2}]$ be the optimal cross-holdings for firm i, $i = 1,2$. From Proposition 1(a) we know that for any arbitrary $s_i \in (0,\frac{1}{2}]$, $\left.\frac{\partial \Gamma_i(r_i,r_j)}{\partial r_i}\right|_{r_i=0,\, s_i=s_i^*} > 0$. Hence, if $s_i = s_i^*$, firm i has an incentive to divest some fraction of its cross-holdings. Therefore, $s_i^* \in (0,\frac{1}{2}]$ cannot be the optimal cross-holdings for firm i. In other words, there does not exist any $s_i \in (0,\frac{1}{2}]$ which can be the optimal cross-holdings for firm i, implying that it is optimal for firm i to divest its cross-holdings fully. ■

Proof of Proposition 3:

From (8), we can infer the following:

(a) If goods are substitutes and strategies are strategic complements, $\frac{\partial \pi_j}{\partial g_i}\frac{\partial g_i^*}{\partial r_i} < 0$ and $\frac{\partial \pi_i}{\partial g_j}\frac{\partial g_j^*}{\partial r_i} < 0$, thus $\frac{\partial Z^*}{\partial r_i} < 0$; hence, there are no incentives for divestment. Moreover, if goods are complements and strategies are strategic complements, $\frac{\partial \pi_j}{\partial g_i}\frac{\partial g_i^*}{\partial r_i} < 0$ and $\frac{\partial \pi_i}{\partial g_j}\frac{\partial g_j^*}{\partial r_i} < 0$, thus $\frac{\partial Z^*}{\partial r_i} < 0$; hence, once more there are no incentives for divestment.

(b) If goods are either substitutes or complements and strategies are strategic substitutes, then $\frac{\partial \pi_j}{\partial g_i}\frac{\partial g_i^*}{\partial r_i} < 0$ and $\frac{\partial \pi_i}{\partial g_j}\frac{\partial g_j^*}{\partial r_i} > 0$, thus the $Sign\left(\frac{\partial Z^*}{\partial r_i}\right)$ is ambiguous.

Now, if firms are symmetric in all respects, thus $s_i = s_j = s$, then from equation (8) we get the following.

$$\left.\frac{\partial Z^*}{\partial r_i}\right|_{r_i=r_j=r\in[0,s]} = \frac{(1-s+r)}{1-s}\left[\frac{\partial \pi_j}{\partial g_i}\frac{\partial g_i^*}{\partial r_i} + \frac{\partial \pi_i}{\partial g_j}\frac{\partial g_j^*}{\partial r_i}\right]$$

$$= \frac{(1-s+r)}{1-s}\left[\frac{\partial \pi_j}{\partial g_i}\frac{\frac{\partial \pi_j}{\partial g_i}\frac{\partial^2 \Omega_j}{\partial g_j^2}}{\widetilde{H}} + \frac{\partial \pi_i}{\partial g_j}\frac{-\frac{\partial \pi_j}{\partial g_i}\frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}}{\widetilde{H}}\right] \text{(see proof of Lemma 1)}$$

$= \frac{(1-s+r)}{(1-s)\widetilde{H}}\left(\frac{\partial \pi_j}{\partial g_i}\right)^2\left[\frac{\partial^2 \Omega_j}{\partial g_j^2} - \frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}\right] < 0$. This is because by symmetry, we have in equilibrium that $\frac{\partial \pi_j}{\partial g_i} = \frac{\partial \pi_i}{\partial g_j}$; moreover, by Assumption A2, we have that $\frac{\partial^2 \Omega_j}{\partial g_j^2} < 0$ and $\left|\frac{\partial^2 \Omega_j}{\partial g_j^2}\right| > \left|\frac{\partial^2 \Omega_j}{\partial g_i \partial g_j}\right|$, with $\frac{\partial^2 \Omega_j}{\partial g_i \partial g_j} = \frac{\partial^2 \Omega_i}{\partial g_i \partial g_j}$ and $\frac{\partial^2 \Omega_j}{\partial g_j^2} = \frac{\partial^2 \Omega_i}{\partial g_i^2}$ in equilibrium. Therefore, if strategies are strategic substitutes and goods are either substitutes or complements, $\left.\frac{\partial Z^*}{\partial r_i}\right|_{r_i = r_j = r \in [0,s]} < 0$, which implies that there are no incentives to divest.

Note that this is also true whenever $\left|\frac{\partial \pi_j}{\partial g_i}\right| \geq \left|\frac{\partial \pi_i}{\partial g_j}\right|$ and $\left((1+s_i) - (1+s_j)(s_i - r_i)\right) \geq \left((1+s_j) - (1+s_i)(s_j - r_j)\right)$, in which case $\frac{\partial Z^*}{\partial r_i} < 0$. ■

Properties of the Follower's Reaction Function:

The first order condition of the follower's stage 4 problem can be written as

$$\frac{\partial \Gamma_2}{\partial g_2} = \frac{1}{1 - (s_1 - \alpha_1 r_1)(s_2 - r_2)} \left[\frac{\partial \pi_2}{\partial g_2} + (s_2 - r_2)\frac{\partial \pi_1}{\partial g_2}\right] = 0 \tag{A3}$$

Let $g_{2,RF}(g_1, r_1, r_2)$ be the solution of (A3). Then

$$\frac{\partial g_{2,RF}}{\partial g_1} = -\frac{\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}}{\frac{\partial^2 \Gamma_2}{\partial g_2^2}} \begin{cases} < 0, if \frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} < 0 \\ > 0, if \frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0 \end{cases}, \text{ since } \frac{\partial^2 \Gamma_2}{\partial g_2^2} < 0 \text{ (by Assumption A2).}$$

Further, note that $\left|\frac{\partial g_{2,RF}}{\partial g_1}\right| < 1$, since $\left|\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}\right| < \left|\frac{\partial^2 \Gamma_2}{\partial g_2^2}\right|$ by Assumption A2. Now, from equation (A3) it follows $g_{2,RF}(g_1, r_1, r_2)$ satisfies $\frac{\partial \pi_2}{\partial g_2} + (s_2 - r_2)\frac{\partial \pi_1}{\partial g_2} = 0$. Therefore, we get: (a) $\frac{\partial g_{2,RF}}{\partial r_1} = 0$, and thus we write, $g_{2,RF}(g_1, r_2)$, and (b) $\frac{\partial g_{2,RF}}{\partial r_2} = \frac{\frac{\partial \pi_1}{\partial g_2}}{\frac{\partial^2 \pi_2}{\partial g_2^2} + (s_2 - r_2)\frac{\partial^2 \pi_1}{\partial g_2^2}} \Rightarrow Sign\left(\frac{\partial g_{2,RF}}{\partial r_2}\right) = -Sign(\frac{\partial \pi_1}{\partial g_2})$, since $\frac{\partial^2 \pi_2}{\partial g_2^2} + (s_2 - r_2)\frac{\partial^2 \pi_1}{\partial g_2^2} < 0$ (by Assumption A2 $\frac{\partial^2 \Gamma_2}{\partial g_2^2} < 0$; also $\frac{1}{1 - (s_1 - \alpha_1 r_1)(s_2 - r_2)} > 0$) ■

Proof of Lemma 2:

The first order condition of the leader's problem, $\max_{g_1} \Gamma_1(g_1, g_2, r_1, r_2)$ subject to the constraint $g_2 = g_{2,RF}(g_1, r_2)$, in stage 3 of the game is,

$$\frac{d\Gamma_1}{dg_1} = \frac{\partial \Gamma_1}{\partial g_1} + \frac{\partial \Gamma_1}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1} = 0 \tag{A4}$$

$$\Rightarrow \frac{d\Gamma_1}{dg_1} = \frac{\partial \pi_1}{\partial g_1} + (s_1 - r_1)\frac{\partial \pi_2}{\partial g_1} + \left[\frac{\partial \pi_1}{\partial g_2} + (s_1 - r_1)\frac{\partial \pi_2}{\partial g_2}\right]\frac{\partial g_{2,RF}}{\partial g_1} = 0 \tag{A5}$$

Notice that $\frac{d\Gamma_1}{dg_1}$ depends on r_2 only via $\frac{\partial g_{2,RF}}{\partial g_1}$. The second order condition of the leader's problem,

$$m \equiv \frac{\partial}{\partial g_1}\left[\frac{\partial \Gamma_1}{\partial g_1}+\frac{\partial \Gamma_1}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right]+\frac{\partial}{\partial g_2}\left[\frac{\partial \Gamma_1}{\partial g_1}+\frac{\partial \Gamma_1}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right]\frac{\partial g_{2,RF}}{\partial g_1}=\left(\frac{\partial^2 \Gamma_1}{\partial g_1^2}+\frac{\partial^2 \Gamma_1}{\partial g_1 \partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right)+\left(\frac{\partial^2 \Gamma_1}{\partial g_2 g_1}+\frac{\partial^2 \Gamma_1}{\partial g_2^2}\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_{2,RF}}{\partial g_1}+\frac{\partial \Gamma_1}{\partial g_2}\frac{\partial^2 g_{2,RF}}{\partial g_1^2}<0,$$ is assumed to be satisfied.

Note that since (i) $\frac{\partial^2 \Gamma_i}{\partial g_i^2}<0$ and $\left|\frac{\partial^2 \Gamma_i}{\partial g_i^2}\right|>\left|\frac{\partial^2 \Gamma_i}{\partial g_i \partial g_j}\right|>\left|\frac{\partial^2 \Gamma_i}{\partial g_j^2}\right|$ by Assumption A2, and (ii) $\left|\frac{\partial g_{2,RF}}{\partial g_1}\right|<1$ (as shown above), we get $\left(\frac{\partial^2 \Gamma_1}{\partial g_1^2}+\frac{\partial^2 \Gamma_1}{\partial g_1 \partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right)+\left(\frac{\partial^2 \Gamma_1}{\partial g_2 g_1}+\frac{\partial^2 \Gamma_1}{\partial g_2^2}\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_{2,RF}}{\partial g_1}<0$. Therefore, $m<0$ whenever $\frac{\partial^2 g_{2,RF}}{\partial g_1^2}=0$, or $\frac{\partial \Gamma_1}{\partial g_2}\frac{\partial^2 g_{2,RF}}{\partial g_1^2}<0$, or $\left|\frac{\partial \Gamma_1}{\partial g_2}\frac{\partial^2 g_{2,RF}}{\partial g_1^2}\right|<\left|\left(\frac{\partial^2 \Gamma_1}{\partial g_1^2}+\frac{\partial^2 \Gamma_1}{\partial g_1 \partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right)+\left(\frac{\partial^2 \Gamma_1}{\partial g_2 g_1}+\frac{\partial^2 \Gamma_1}{\partial g_2^2}\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_{2,RF}}{\partial g_1}\right|$.

Then, using the implicit function theorem, we get from (A5),

$$\frac{\partial g_1^L}{\partial r_1}=\frac{\frac{\partial \pi_2}{\partial g_1}+\frac{\partial \pi_2}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial g_1}}{m} \tag{A6}$$

By (A3), $\frac{\partial \pi_2}{\partial g_2}=\ -(s_2-r_2)\ \frac{\partial \pi_1}{\partial g_2}$. Thus,

$$\frac{\partial g_1^L}{\partial r_1}=\frac{\frac{\partial \pi_2}{\partial g_1}-(s_2-r_2)\frac{\partial \pi_1}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial g_1}}{m}. \tag{A7}$$

Now, by the second order condition of the leader's maximization problem, $m<0$; also, $\frac{\partial \pi_j}{\partial g_i}<(>)0)$ if goods are substitutes (complements), and $\frac{\partial g_{2,RF}}{\partial g_1}<(>)0$ if strategies are strategic substitutes (strategic complements). Therefore, we get,

1. If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}<0$ and $\frac{\partial \pi_i}{\partial g_j}<0$, then $\frac{\partial g_1^L}{\partial r_1}>0$.
2. If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}<0$ and $\frac{\partial \pi_i}{\partial g_j}>0$, then, $\frac{\partial g_1^L}{\partial r_1}<0$.
3. If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}>0$ and $\frac{\partial \pi_i}{\partial g_j}<0$, then $\frac{\partial g_1^L}{\partial r_1}>0$ if and only if $\frac{\partial \pi_2}{\partial g_1}+\frac{\partial \pi_2}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial g_1}<0$, or else, $\frac{\partial g_{2,RF}}{\partial g_1}<-\frac{\frac{\partial \pi_2}{\partial g_1}}{\frac{\partial \pi_2}{\partial g_2}}=\left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=const}$ (since by (A3), $\frac{\partial \pi_2}{\partial g_2}=-(s_2-r_2)\frac{\partial \pi_1}{\partial g_2}>0$), with $\left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=constant}$ the slope of firm 2's iso-profit curve in the $g_1 g_2$-plane.
4. If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2}>0$ and $\frac{\partial \pi_i}{\partial g_j}>0$, then $\frac{\partial g_1^L}{\partial r_1}<0$ if and only if $\frac{\partial \pi_2}{\partial g_1}+\frac{\partial \pi_2}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial g_1}>0$, or esle, $\frac{\partial g_{2,RF}}{\partial g_1}<-\frac{\frac{\partial \pi_2}{\partial g_1}}{\frac{\partial \pi_2}{\partial g_2}}=\left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=const}$ (since by (A3), $\frac{\partial \pi_2}{\partial g_2}=-(s_2-r_2)\frac{\partial \pi_1}{\partial g_2}<0$.)

To be able to draw clear conclusions in cases 3 and 4, we first prove the following Lemma.

Lemma 3: *If* $\frac{\partial^2 \pi_i}{\partial g_j \partial g_i} > 0$, *then* $\frac{\partial g_{2,RF}}{\partial g_1} < \left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=const}$.

Proof of Lemma 3: Remember that $g_{2,RF}(g_1, r_2)$ is the solution of (A3). Letting $g_{2,\pi_2}(g_1) = argmax_{g_2}\, \pi_2(g_1, g_2)$, we have,
$\frac{\partial g_{2,\pi_2}}{\partial g_1} = \frac{\frac{\partial^2 \pi_2}{\partial g_1 \partial g_2}}{-\frac{\partial^2 \pi_2}{\partial g_2^2}} < \frac{\frac{\partial^2 \pi_2}{\partial g_2 \partial g_1} + (s_2 - r_2)\frac{\partial^2 \pi_1}{\partial g_2 \partial g_1}}{-\frac{\partial^2 \pi_2}{\partial g_2^2} - (s_2 - r_2)\frac{\partial^2 \pi_1}{\partial g_2^2}} = \frac{\partial g_{2,RF}}{\partial g_1}$. The former inequality holds as $s_2 \geq r_2 \geq 0$, $\frac{\partial^2 \pi_i}{\partial g_j \partial g_i} > 0$, $-\frac{\partial^2 \pi_2}{\partial g_2^2} > 0$, and $\left|\frac{\partial^2 \pi_i}{\partial g_i^2}\right| > \left|\frac{\partial^2 \pi_i}{\partial g_j \partial g_i}\right| > \left|\frac{\partial^2 \pi_i}{\partial g_j^2}\right|$ (by Assumption A2). Further, by (A3) the latter equality is true. Note that if $\frac{\partial^2 \pi_i}{\partial g_i \partial g_j} > 0$, then $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0$ and thus, $\frac{\partial g_{2,\pi_2}}{\partial g_1} > 0$.

Hence, when $\frac{\partial^2 \pi_i}{\partial g_i \partial g_j} > 0$, we have $0 < \frac{\partial g_{2,\pi_2}}{\partial g_1} < \frac{\partial g_{2,RF}}{\partial g_1}$, and the following property holds.

Property 1: *Both $g_{2,RF}(g_1, r_2)$ and $g_{2,\pi_2}(g_1)$ are positively sloped and $g_{2,RF}(g_1, r_2)$ is steeper than $g_{2,\pi_2}(g_1)$ in the $g_1 g_2$-plane.*

Further, since $g_{2,RF}(g_1, r_2)$ is given by $\frac{\partial \Gamma_2}{\partial g_2} = \frac{1}{1-(s_1 - \alpha_1 r_1)(s_2 - r_2)}\left[\frac{\partial \pi_2}{\partial g_2} + (s_2 - r_2)\frac{\partial \pi_1}{\partial g_2}\right] = 0$ and $g_{2,\pi_2}(g_1)$ is given by $\frac{\partial \pi_2}{\partial g_2} = 0$, we get,
$\left.\frac{\partial \Gamma_2}{\partial g_2}\right|_{g_2 = g_{2,\pi_2}} = \frac{1}{1-(s_1 - \alpha_1 r_1)(s_2 - r_2)}\left[(s_2 - r_2)\frac{\partial \pi_1}{\partial g_2}\right] > (<) 0 \Leftrightarrow \frac{\partial \pi_1}{\partial g_2} > (<) 0$. This implies the following property.

Property 2: *For any given g_1, $g_{2,RF} < g_{2,\pi_2}$ when $\frac{\partial \pi_1}{\partial g_2} < 0$, and $g_{2,RF} > g_{2,\pi_2}$ when $\frac{\partial \pi_1}{\partial g_2} > 0$.*

Next, $\pi_2(g_1, g_2) = const$ implies that $\frac{\partial \pi_2}{\partial g_2}\left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2 = const.} + \frac{\partial \pi_2}{\partial g_1} = 0 \Rightarrow \left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2 = const.} = -\frac{\frac{\partial \pi_2}{\partial g_1}}{\frac{\partial \pi_2}{\partial g_2}} \Rightarrow \left[\left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2 = const.} \text{at } g_2 = g_{2,RF}\right] = -\frac{\frac{\partial \pi_2}{\partial g_1}}{-(s_2 - r_2)\frac{\partial \pi_1}{\partial g_2}} = \frac{\frac{\partial \pi_2}{\partial g_1}}{(s_2 - r_2)\frac{\partial \pi_1}{\partial g_2}} > 0$, regardless of whether $\frac{\partial \pi_i}{\partial g_j} > 0$ or $\frac{\partial \pi_i}{\partial g_j} < 0$. That is, at each point on $g_{2,RF}(g_1, r_2)$, the iso-profit curves $\pi_2(g_1, g_2) = const$ are positively sloped.

Property 3: *Regardless of whether $g_{2,RF} < (>) g_{2,\pi_2}$, i.e., whether $\frac{\partial \pi_1}{\partial g_2} < (>) 0$, at each point on $g_{2,RF}(g_1, r_2)$, the iso-profit curves $\pi_2(g_1, g_2) = const$ are positively sloped.*

Further, note that the following property is always true.

Property 4: $\left.\frac{dg_2}{dg_1}\right|_{\pi_2=const} = \infty$ *at every point on* $g_{2,\pi_2}(g_1)$*, i.e., the tangent of the iso-profit curves* $\pi_2(g_1, g_2) = const$*, is a vertical line at each point on* $g_{2,\pi_2}(g_1)$ *in the* $g_1 g_2$*-plane.*

Now, from Properties 1-4 and Assumptions A1 and A2, it follows that $\frac{\partial g_{2,RF}}{\partial g_1} < \left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=const}$ whenever $\frac{\partial^2 \pi_i}{\partial g_i \partial g_j} > 0$. That is, if $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0$, we have $\frac{\partial g_{2,RF}}{\partial g_1} < \left.\frac{\partial g_2}{\partial g_1}\right|_{\pi_2=const}$, since by Assumption A3, $\frac{\partial^2 \pi_i}{\partial g_i \partial g_j} > 0$ if and only if $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0$. ■

By Lemma 3, we infer that,

(i) If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0$ and $\frac{\partial \pi_i}{\partial g_j} < 0$, then $\frac{\partial g_1^L}{\partial r_1} > 0$.

(ii) If $\frac{\partial^2 \Gamma_2}{\partial g_1 \partial g_2} > 0$ and $\frac{\partial \pi_i}{\partial g_j} > 0$, then $\frac{\partial g_1^L}{\partial r_1} < 0$.

In sum, $\frac{\partial g_1^L}{\partial r_1} > 0\ (< 0)$, if goods are substitutes (complements), regardless of the nature of firms' strategic variables.

Next, considering (A4) and applying the implicit function theorem we get,

$$\frac{\partial g_1^L}{\partial r_2} = -\frac{1}{m}\left[\frac{\partial\left(\frac{d\Gamma_1}{dg_1}\right)}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial r_2}\right].$$

This implies that,

$Sign\left(\frac{\partial g_1^L}{\partial r_2}\right) = Sign\left(\frac{\partial\left(\frac{d\Gamma_1}{dg_1}\right)}{\partial g_2}\right) Sign\left(\frac{\partial g_{2,RF}}{\partial r_2}\right) = -Sign\left(\frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1}\right) Sign\left(\frac{\partial \pi_1}{\partial g_2}\right)$. This is because,

(a) $m < 0$ by the second-order condition of the leader's problem,

(b) $Sign\left(\frac{\partial g_{2,RF}}{\partial r_2}\right) = -Sign\left(\frac{\partial \pi_1}{\partial g_2}\right)$ and

(c) $\frac{\partial\left(\frac{d\Gamma_1}{dg_1}\right)}{\partial g_2} = \frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1} + \frac{\partial^2 \Gamma_1}{\partial g_2^2}\frac{\partial g_{2,RF}}{\partial g_1} \Rightarrow Sign\left(\frac{\partial\left(\frac{d\Gamma_1}{dg_1}\right)}{\partial g_2}\right) = Sign\left(\frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1}\right)$, since by Assumption A2 $\left|\frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1}\right| > \left|\frac{\partial^2 \Gamma_1}{\partial g_2^2}\right|$ and $\left|\frac{\partial g_{2,RF}}{\partial g_1}\right| < 1$. Therefore, we obtain,

$$\frac{\partial g_1^L}{\partial r_2}\begin{cases} > 0, if\ \frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1} > (<) 0\ \ and\ \frac{\partial \pi_1}{\partial g_2} < (>) 0 \\ < 0, if\ \frac{\partial^2 \Gamma_1}{\partial g_2 \partial g_1} > (<) 0\ \ and\ \frac{\partial \pi_1}{\partial g_2} > (<) 0 \end{cases}.$$

Now, since $g_2^F = g_2^F(g_1^L(r_1, r_2), r_2)$, we can write $\frac{\partial g_2^F}{\partial r_1} = \frac{\partial g_{2,RF}}{\partial g_1}\frac{\partial g_1^L}{\partial r_1}$. Since (i) $\frac{\partial g_{2,RF}}{\partial g_1} < (>) 0$ if $\frac{\partial^2 \Gamma_2}{\partial g_2 \partial g_1} < (>) 0$ and (ii) Lemma 2(a) holds true regarding $\frac{\partial g_1^L}{\partial r_1}$, we get,

$$\frac{\partial g_2^F}{\partial r_1}\begin{cases} > 0, if\ \frac{\partial^2\Gamma_2}{\partial g_2 \partial g_1} > (<) 0\ \ and\ \frac{\partial \pi_1}{\partial g_2} < (>) 0 \\ < 0, if\ \frac{\partial^2\Gamma_2}{\partial g_2 \partial g_1} > (<) 0\ \ and\ \frac{\partial \pi_1}{\partial g_2} > (<) 0 \end{cases}.$$

Finally, $\frac{\partial g_2^F}{\partial r_2} = \frac{\partial g_{2,RF}}{\partial r_2} + \frac{\partial g_{2,RF}}{\partial g_1}\frac{\partial g_1^L}{\partial r_2}$. Since (i) $\frac{\partial g_{2,RF}}{\partial r_2} > (<) 0$ if $\frac{\partial \pi_2}{\partial g_1} < (>) 0$, (ii) $\frac{\partial g_{2,RF}}{\partial g_1} < (>) 0$ if $\frac{\partial^2\Gamma_2}{\partial g_2 \partial g_1} < (>) 0$, and (iii) Lemma 2(b) holds true regarding $\frac{\partial g_1^L}{\partial r_2}$, it follows that

$$\frac{\partial g_2^F}{\partial r_2}\begin{cases} > 0, if\ \frac{\partial \pi_i}{\partial g_j} < 0 \\ < 0, if\ \frac{\partial \pi_i}{\partial g_j} > 0 \end{cases},$$

regardless of whether strategies are strategic substitutes or strategic complements. ■

Proof of Proposition 4:

Note first that,

$$\frac{d\Gamma_1^L(\cdot)}{dr_1} = \frac{1}{1-s_1 s_2}\left[\frac{d\pi_1^L}{dr_1} + s_1\frac{d\pi_2^F}{dr_1}\right] = \frac{1}{1-s_1 s_2}\left[\left(\frac{\partial \pi_1^L}{\partial g_1^L} + s_1\frac{\partial \pi_2^F}{\partial g_1^L}\right)\frac{\partial g_1^L}{\partial r_1} + \left(\frac{\partial \pi_1^L}{\partial g_2^F} + s_1\frac{\partial \pi_2^F}{\partial g_2^F}\right)\frac{\partial g_2^F}{\partial r_1}\right].$$

Using equations (A3) and (A5) and after some algebraic manipulations, we get,

$$\frac{d\Gamma_1^L(\cdot)}{dr_1} = \left[\frac{r_1}{1-s_1 s_2}\frac{\partial \pi_2^F}{\partial g_1}\right]\frac{\partial g_1^L}{\partial r_1} + \left[-\frac{1-(s_1-r_1)(s_2-r_2)}{1-s_1 s_2}\frac{\partial \pi_1^L}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right]\frac{\partial g_1^L}{\partial r_1} + \left[\frac{1-s_1(s_2-r_2)}{1-s_1 s_2}\frac{\partial \pi_1^L}{\partial g_2}\right]\frac{\partial g_2^F}{\partial r_1} \quad \text{(A8)}$$

$$= \frac{1}{1-s_1 s_2}\left[r_1\frac{\partial \pi_2^F}{\partial g_1}\frac{\partial g_1^L}{\partial r_1}\right] + \frac{1}{1-s_1 s_2}\left[-r_1(s_2-r_2)\frac{\partial \pi_1^L}{\partial g_2}\frac{\partial g_{2,RF}}{dg_1}\right]\frac{\partial g_1^L}{\partial r_1},$$

since $\frac{\partial g_2^F}{\partial r_1} = \frac{\partial g_{2,RF}}{\partial g_1}\frac{\partial g_1^L}{\partial r_1}$. Therefore,

$$\frac{d\Gamma_1^L(\cdot)}{dr_1} = \frac{1}{1-s_1 s_2}\ r_1\ \left(\frac{\partial g_1^L}{\partial r_1}\right)^2 m < 0,$$

since by equation (A7), $\frac{\partial g_1^L}{\partial r_1} = \frac{\left[\frac{\partial \pi_2}{\partial g_1} - (s_2-r_2)\frac{\partial \pi_1}{\partial g_2}\ \frac{\partial g_{2,RF}}{\partial g_1}\right]}{m}$, and by the second order condition of the leader's maximization problem $m < 0$.

Note second that,

$$\frac{d\Gamma_2^F(\cdot)}{dr_2} = \frac{1}{1-s_1 s_2}\left[\frac{d\pi_2^F}{dr_2} + s_2\frac{d\pi_1^L}{dr_2}\right] = \frac{1}{1-s_1 s_2}\left[\left(\frac{\partial \pi_2^F}{\partial g_1^L} + s_2\frac{\partial \pi_1^L}{\partial g_1^L}\right)\frac{\partial g_1^L}{\partial r_2} + \left(\frac{\partial \pi_2^F}{\partial g_2^F} + s_2\frac{\partial \pi_1^L}{\partial g_2^F}\right)\frac{\partial g_2^F}{\partial r_2}\right].$$

Using equations (A3) and (A5), we get,

$$\frac{d\Gamma_2^F(\cdot)}{dr_2} = \frac{1}{1-s_1 s_2}\left[r_2\frac{\partial \pi_1^L}{\partial g_2^F}\frac{\partial g_2^F}{\partial r_2} + \left(\frac{\partial \pi_2^F}{\partial g_1^L} - s_2(s_1-r_1)\frac{\partial \pi_2^F}{\partial g_1^L} - s_2\left[\frac{\partial \pi_1^L}{\partial g_2^F} + (s_1-r_1)\frac{\partial \pi_2^F}{\partial g_2^F}\right]\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_1^L}{\partial r_2}\right]$$

$$= \frac{1}{1-s_1 s_2}\left[r_2\frac{\partial \pi_1^L}{\partial g_2^F}\frac{\partial g_2^F}{\partial r_2} + \left(\frac{\partial \pi_2^F}{\partial g_1^L} - s_2(s_1-r_1)\frac{\partial \pi_2^F}{\partial g_1^L} - s_2\left[\frac{\partial \pi_1^L}{\partial g_2^F} - (s_1-r_1)(s_2-r_2)\frac{\partial \pi_1^L}{\partial g_2^F}\right]\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_1^L}{\partial r_2}\right]$$

$$= \frac{1}{1-s_1s_2}\left[r_2\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_2^F}{\partial r_2} + \left(\left(1-s_2(s_1-r_1)\right)\frac{\partial\pi_2^F}{\partial g_1^L} - s_2[1-(s_1-r_1)(s_2-r_2)]\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_1^L}{\partial r_2}\right]$$

$$= \frac{1}{1-s_1s_2}\left[r_2\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_2^F}{\partial r_2} + \left(1-s_2(s_1-r_1)\right)\left(\frac{\partial\pi_2^F}{\partial g_1^L} - (s_2-r_2)\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_{2,RF}}{\partial g_1}\right)\frac{\partial g_1^L}{\partial r_2} - r_2\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_{2,RF}}{\partial g_1}\frac{\partial g_1^L}{\partial r_2}\right]$$

$$= \frac{1}{1-s_1s_2}\left[r_2\frac{\partial\pi_1^L}{\partial g_2^F}\frac{\partial g_{2,RF}}{\partial r_2} + \left(1-s_2(s_1-r_1)\right)\frac{\partial g_1^L}{\partial r_1}\frac{\partial g_1^L}{\partial r_2}m\right],$$

since $\frac{\partial g_2^F}{\partial r_2} = \frac{\partial g_{2,RF}}{\partial r_2} + \frac{\partial g_{2,RF}}{\partial g_1}\frac{\partial g_1^L}{\partial r_2}$ and by equation (A7), $\frac{\partial g_1^L}{\partial r_1} = \frac{\left[\frac{\partial\pi_2}{\partial g_1} - (s_2-r_2)\frac{\partial\pi_1}{\partial g_2}\frac{\partial g_{2,RF}}{\partial g_1}\right]}{m}$. Hence,

$$\left.\frac{d\Gamma_2^F(\cdot)}{dr_2}\right|_{r_2=0} = \frac{1-s_2(s_1-r_1)}{1-s_1s_2}\frac{dg_1^L}{dr_1}\frac{dg_1^L}{dr_2}m\ .$$

Then, since $\frac{1-s_2(s_1-r_1)}{1-s_1s_2} > 0$ and $m < 0$, $Sign\left(\left.\frac{d\Gamma_2^F(\cdot)}{dr_2}\right|_{r_2=0}\right) = -Sign\left(\frac{\partial g_1^L}{\partial r_1}\frac{\partial g_1^L}{\partial r_2}\right)$.

Now, from Lemma 2, we have

1. $\frac{\partial g_1^L}{\partial r_1} > 0\ (< 0)$, if $\frac{\partial \pi_i}{\partial g_j} < 0 (> 0)$ regardless of whether $\frac{\partial^2\Gamma_i}{\partial g_i \partial g_j} > 0$ or < 0.
2. $\frac{\partial g_1^L}{\partial r_2}\begin{cases} > 0, if\ \frac{\partial^2\Gamma_1}{\partial g_2\partial g_1} > (<) 0 \text{ and } \frac{\partial\pi_1}{\partial g_2} < (>) 0 \\ < 0, if\ \frac{\partial^2\Gamma_1}{\partial g_2\partial g_1} > (<) 0 \text{ and } \frac{\partial\pi_1}{\partial g_2} > (<) 0 \end{cases}$.

Therefore, it follows that $\left.\frac{d\Gamma_2^F(\cdot)}{dr_2}\right|_{r_2=0}\begin{cases} > 0, if\ \frac{\partial^2\Gamma_1}{\partial g_2\partial g_1} < 0 \\ < 0, if\ \frac{\partial^2\Gamma_1}{\partial g_2\partial g_1} > 0 \end{cases}$. ■

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