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ABSTRACT

We study entry in a differentiated-product Bertrand industry in which firms are privately informed about their marginal costs. We show that policies that facilitate entry, such as per-unit subsidies to entrants increase expected output and total welfare. Under Bayes–Bertrand competition, firms condition their pricing decisions on the expected costs of their rivals rather than on realized costs. In this environment, facilitating entry for relatively inefficient types raises the expected output of inframarginal firms and incumbents, owing to the strategic complementarity of prices. When all firms simultaneously decide whether to enter, it is optimal to allow all potential entrants to participate.

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We study entry in a differentiated-product Bertrand industry in which firms are privately informed about their marginal costs. We show that policies that facilitate entry, such as per-unit subsidies to entrants increase expected output and total welfare. Under Bayes–Bertrand competition, firms condition their pricing decisions on the expected costs of their rivals rather than on realized costs. In this environment, facilitating entry for relatively inefficient types raises the expected output of inframarginal firms and incumbents, owing to the strategic complementarity of prices. When all firms simultaneously decide whether to enter, it is optimal to allow all potential entrants to participate.

1 Introduction

The relationship between entry barriers and social welfare has been extensively studied in oligopoly theory. Mankiw and Whinston (1986) show that a business-stealing effect leads to excessive entry in models with fixed entry costs. Their analysis, however, assumes that firms' marginal costs are common knowledge. While this assumption is not implausible, it abstracts from situations in which firms possess private information about their costs. Incorporating such informational asymmetries raises new questions about the efficiency of entry.

Khezr and Menezes (2021) study entry in a price competition model with private costs but homogeneous goods. They show that both excessive and insufficient entry can arise depending on the realization of costs, however are restricted by a knife-edge equilibrium in which the lowest-cost firm captures the entire market, on account of homogenous goods. More recently, Bisceglia et al. (2024) analyze a Bayes–Cournot model and provide a rationale for raising entry barriers. In their setting, incumbents respond to the expected behavior of entrants rather than their realized types, which can generate a positive first-order welfare effect of restricting entry. While this extends the insights of Mankiw and Whinston (1986) to environments with private information, the analysis is confined to quantity competition.

The closest paper to ours is Ghosh and Zincenko (2025), who study price competition with private marginal costs in a Salop framework. They derive sufficient conditions under which entry

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is excessive and argue that these conditions may fail for a range of cost distributions, in which case free entry is optimal. In contrast, we provide a complete characterization of insufficient entry and study the policy instruments required to implement the efficient outcome in a Perfect Bayesian Equilibrium. Bertoletti and Etro (2024) also document insufficient entry in Bertrand models of monopolistic competition with differentiated products, but assume that marginal costs are common knowledge. Our framework extends this analysis to settings with asymmetric information.

We study entry in a Bayes–Bertrand model with differentiated products. A key feature of the environment is that firms base their pricing decisions on the expected costs of their rivals, conditional on entry, rather than on realized costs. We first consider a setting with $N-1$ incumbents and a single potential entrant. We show that facilitating entry through per-unit subsidies, which allow relatively inefficient types to enter, can have a positive first-order effect on the entrant's expected output, aggregate output, and total welfare, under mild conditions on product differentiation and the cost distribution. The mechanism is straightforward. As entry is facilitated, incumbents expect the marginal entrant to be less efficient and adjust their prices upward. Given that prices are strategic complements, this induces higher prices and output for inframarginal entrant types. This effect would not arise if inefficient types could not enter. In contrast, lump-sum subsidies do not generate similar effects.

We further characterize the per-unit subsidy required to implement the output-maximizing cutoff and extend the analysis to a setting in which all N firms incur a symmetric entry cost. The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes the entry decision of a single entrant. Section 4 extends the analysis to simultaneous entry by all firms. Section 5 concludes. Proofs are collected in the Appendix.

2 Model

2.1 Setup

We closely follow Bisceglia et al. (2024) with two key modifications: firms compete in prices rather than quantities and demand is derived from a differentiated products framework. Consider a set of $N \geq 2$ potential entrants (denoted by $i = 1, ..., N$), each producing a differentiated product in an industry with Bertrand competition characterized by a linear direct demand à la Singh and Vives (1984):

$$q_i = a - bp_i + d \sum_{j \neq i} p_j,$$

where p_i and q_i denote firm i's price and quantity choice. The parameters $b > 0$ and $d > 0$ capture the own and cross price effects, respectively. Throughout, we assume that $b > d$, so that own price effects dominate cross price effects. It is convenient to reparametrize the model by defining $\frac{d}{b} = \gamma \in (0, 1)$, which indexes an inverse measure of product differentiation with lower values of γ correspond to higher degrees of product differentiation.

To ensure non-negative prices and quantities in equilibrium, we impose the following assumption on the demand system:

Assumption 1. *Products are sufficiently differentiated.*

$$2b > d(N-1)$$

Or equivalently, $\gamma < \frac{2}{N-1}$. *Later, we will assume the stronger condition* $b > d(N-1)$ *for welfare analysis.*

Marginal costs of each firm (c_i) are ex ante uncertain and independently drawn from the bounded support $[0, \bar{c_i}]$, with a smooth cdf $F_i(c_i)$, density $f_i(c_i)$ and unconditional mean $E[c_i] = \tilde{c_i}$.

The timing of the game is as follows:

At t = 1, each potential entrant privately observes its marginal cost c_i, and decides to sink an entry cost k_i to enter the market.

At t = 2, entry decisions are observed, and all firms who are active in the market engage in a Bayes-Bertrand competition. Productions costs remain private information.

We denote the entry strategy of firm i by $x_i(c_i) \in \{0, 1\}$, taking 1 if firm i enters. The appropriate solution concept to this model is a Perfect Bayesian Equilibrium. A PBE here is a collection of: (i) a tuple of entery and pricing strategies $\{(x_i(c_i))_{i=1,\dots,N}\ ,\ (p_i(c_i, x_{-i}))_{i=1,\dots,N}\}$, where x_{-i} is the vector of rivals' entry decisions. (ii) for each entrant i, the rivals' beliefs on its cost, given its entry decision. The strategy profile should be sequentially rational given beliefs, and beliefs must be compatible with the strategy profile. Beliefs are obtained using Bayes rule, whenever possible, given entry strategies. [1]

2.2 Equilibrium

We solve the model by backward induction. Each entrant i solves

$$\max_{p_i \geq 0} [p_i - c_i][a - bp_i + d \sum_{j \neq i} p_j^e(x_j)]$$

with $p_j^e(x_j) = E[p_j^*(c_j, x_j | x_j)]$ being the price that firm i expects firm j to charge conditional on its entry decision. Denote by n the number of entrants in the market. Differentiating with respect to p_i, the first order condition yields firm i's best-response function

$$p_i^*(c_i, x_{-i}) = \frac{1}{2b}[a + bc_i + d \sum_{j \neq i} p_j^e(x_j)]. \tag{1}$$

Upon taking expectation given $x_i = 1$ on both sides of (1) and aggregating we have:

$$\sum_{j \neq i} p_j^e(x_j) = \frac{a(n-1)}{2b - d(n-1)} + \frac{1}{2b + d} \left[\frac{2b^2 \sum_{j \neq i}^{n} \hat{c}_j + bd(n-1)\hat{c}_i}{2b - d(n-1)} \right] \tag{2}$$

where

$$\hat{c}_i = E[c_i | x_i = 1] = \int_{\{c_i : x_i^*(c_i) = 1\}} c_i dF(c_i | x_i = 1) \tag{3}$$

denotes the conditional expectation of firm i's cost from its rivals' standpoint, conditional on the fact that it decided to enter the market.

From (1), given rivals' entry decisions, firm i's expected profit after entry is

$$\pi_i^*(c_i, x_{-i}) = [p_i^*(c_i, x_{-i}) - c_i][a - bp_i^*(c_i, x_{-i}) + d \sum_{j \neq i} p_j^e(x_j)] = b(p_i^*(c_i, x_{-i}) - c_i)^2 = \frac{q_i^*(c_i, x_{-i})^2}{b} \tag{4}$$

[1] Similar to Bisceglia et al. (2024), we consider only cases where any firm enters with a positive probability, such that there is no need to specify off-path beliefs.

Now moving to stage I, each firm i enters the market if and only if its expected profit exceeds the setup cost:

$$E[\pi_i^*(c_i, x_{-i})] > k_i,$$

where the expectation is taken with respect to rivals' entry decisions. If $q_i^*(.)$ is strictly decreasing in c_i for all x_{-i}, then the expected profit also falls in c_i. Then, for any candidate PBE of the game, we can define cutoffs $(c_i^*)_{i=1,\dots,N}$ such that $x_i^*(c_i) = 1$ if and only if $c_i < c_i^*$. Then, using Bayes rule, rival's beliefs on firm i's costs, conditional on entry, are such that

$$\hat{c}_i(c_i^*) = E[c_i | c_i < c_i^*]. \tag{5}$$

Firms respond to the expected cost of their rivals, and not the actual cost. Each firm updates its belief about the average cost of an entrant upon observing entry at different cutoffs.

To ensure $q_i^*(.)$ is strictly decreasing in c_i, we impose our second assumption on the cost distribution:

Assumption 2. *The expected costs are not too sensitive to changes in the cutoff.*

$$\frac{\partial \hat{c}_i(c_i^*)}{\partial c_i^*} < \frac{(2b+d)(2b-d(N-1))}{d^2(N-1)} \quad \forall\, c_i^*$$

While this assumption may appear restrictive, it is fairly naive in case of well behaved cost distributions. Given that $2b > d(N-1)$, the upper threshold $\frac{(2b+d)(2b-d(N-1))}{d^2(N-1)}$ is greater than one. Then, Assumption 2 gets violated only when the change in expected costs with change in the cutoff is greater than one. For smooth and regular cost distributions, this never goes through. [2]

3 Game with one potential entrant

Suppose that N-1 firms (incumbents) are already in the market and do not face any entry costs. For all incumbents $j \neq i$, their expected costs are equal to the unconditional means: $\hat{c}_j = \tilde{c}_j$ because they do not have to enter and their presence does not signal anything about their costs.

Now, firm i enters if and only if $\pi_i^*(c_i, \mathbf{1}) > k$. The equilibrium cutoff c_i^* is then obtained as a solution of:

$$p_i^*(c_i^*, \mathbf{1}) - c_i^* = \sqrt{\frac{k}{b}}. \tag{6}$$

Throughout this section, entry decision is made only by firm i. Thus, we can simplify notation by dropping i subscripts appropriately. For example, we just write k instead of k_i. The equilibrium cutoff c^* is defined as follows:

Lemma 1. *Let c^* be the unique solution of:*

$$\frac{1}{2}\left[\frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n}\tilde{c}_j + d^2(N-1)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))} - c^*\right] = \sqrt{\frac{k}{b}} \tag{7}$$

with c^* being decreasing in k. Then, for any given k, the equilibrium is:

- For every $c_i < c^*$, firm i enters the market and charges

$$p_i^*(c_i, c^*) = \frac{1}{2}\left[\frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n}\tilde{c}_j + d^2(N-1)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))} + c_i\right]. \tag{8}$$

[2] For example, Let $c_i \sim U[0, \bar{c}_i]$. Then, $\hat{c}_i(c_i^*) = \frac{c_i^*}{2}$ and $\frac{\partial \hat{c}_i(c_i^*)}{\partial c_i^*} = \frac{1}{2} < 1$.

Every incumbent with cost c_j charges

$$p_j^*(c_j, c^*) = \frac{1}{2}\left[\frac{4ab + 2ad + 2bd\left[\sum_{h\notin\{i,j\}} \tilde{c_h} + \hat{c}(c^*)\right] + d^2(N-1)\tilde{c_j}}{(2b+d)(2b-d(N-1))} + c_j\right]. \tag{9}$$

- For every $c_i \geq c^*$, firm i does not enter and every incumbent j with cost c_j charges

$$p_j^*(c_j) = \frac{1}{2}\left[\frac{4ab + 2ad + 2bd\sum_{h\neq j} \tilde{c_h} + d^2(N-2)\tilde{c_j}}{(2b+d)(2b-d(N-2))} + c_j\right]. \tag{10}$$

Another result that should be established before moving on to the expected output analysis is the relationship between beliefs $\hat{c}$ and the cutoff c^*:

Lemma 2. *As entry becomes relatively easier, that is, as c^* rises, the incumbents' belief of the marginal cost of the entrant also rises:*

$$\frac{\partial \hat{c}(c^*)}{\partial c^*} = \frac{f_i(c^*)}{F_i(c^*)}\left[c^* - \hat{c}(c^*)\right] > 0 \tag{11}$$

3.1 Expected output

In this section, we will describe how maximization of expected entrant's and aggregate output requires facilitating entry. We will show that both these outputs are maximised at a cutoff larger than the cutoff under unrestricted entry- that is, the cutoff c_0^* that solves (7) for $k=0$, or equivalently such that the entrant's price in (8) equals their marginal cost and their quantity equals zero: $q_i^*(c_0^*, c_0^*) = 0$.

Expected entrant's output. For any given cutoff c^*, firm i's expected output is:

$$\begin{aligned} q_i^*(c^*) &= F_i(c^*)\,\mathbb{E}[q_i^*(c_i, c^*) \mid c_i < c^*] \\ &= F_i(c^*)\,b\left[\frac{2ab + ad + bd\sum_{j\neq i}^n \tilde{c}_j + (d^2(N-1) - 2b^2 + bd(N-1) - bd)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))}\right] \end{aligned}$$

Next, we derive the first-order condition for the above quantity to derive the expression for the cutoff which maximises expected entrant's output ($p^\dagger$). Whether maximising it requires entry faciliation, depends upon the relation of $p^\dagger$ to the cutoff under unrestricted entry (c_0^*). We can show that:

Proposition 1. *Facilitating entry increases the entrant's expected output. The cost cutoff which maximizes $q_i^*(c^*)$ is*

$$c^* = p^\dagger = \frac{2ab + ad + bd\sum_{j\neq i}\tilde{c}_j}{2b^2 + bd - d^2(N-1) - bd(N-1)} > c_0^*.$$

The intuition of this result is as follows. The marginal entrant type c_0^* entering at the cutoff c_0^* produces zero output. Any entrant with costs greater than this type cannot enter at this cutoff, as they are not efficient enough to charge a price greater than their own marginal cost and produce a positive output. However, allowing some of these inefficient types ($c_0^* < c_i < p^\dagger$) to enter the market has a positive externality on the inframarginal entrant types ($c_i < c_0^*$). As the cutoff is raised beyond c_0^*, the incumbents expect the average entrant type to be more inefficient. This allows them to charge a higher price and prices being strategic complements, this allows the entrant in turn to charge a higher price and in equilibrium, produce a higher quantity. This effect cannot be scaled endlessly due to the opposing effect of higher cost entrants being less efficient and producing lower quantities.

The immediate concern then is to construct a credible mechanism for raising the equilibrium cutoff beyond the free entry cutoff. For a PBE, the incumbents should be able to rationally hold the belief that these otherwise inefficient types will now be able to enter the market. Reducing k does increase c^*, however raising it beyond c_0^* requires negative values of k, which we do not allow. While these can be interpreted as a net lumpsum subsidy, it cannot credibly raise the PBE cutoff. The reason is as follows. A lumpsum subsidy does not necessarily result in a positive output for inefficient firms. While the subsidy would allow firms who would otherwise make operational losses to cover them and enter the market, rational firms can simply take the subsidy and choose to produce zero output. The incumbents will fully anticipate this and will not consider entry by the inefficient types to be a credible threat, and accordingly will not raise the prices, thus blocking the positive externality on the inframarginal types. We deal with this issue by modelling a per unit subsidy.

Per unit subsidy. Let s be the per unit subsidy given to the potential entrant. Now, our cutoff condition (7) is updated as follows:

$$\frac{1}{2}\left[\frac{4ab+2ad+2bd\sum_{j\neq i}^{n}\tilde{c_j}+d^2(N-1)(\hat{c}(c^*)-s)}{(2b+d)(2b-d(N-1))}-(c^*-s)\right]=\sqrt{\frac{k}{b}} \tag{12}$$

where c^* decreases in k and increases in s. Keeping $k=0$, $c^*=p^\dagger$ can be implemented using a per unit subsidy:

$$s^*=\frac{(2b+d)(2b-d(N-1))}{2b(2b-d(N-1))+2d(b-d(N-1))}\left(p^\dagger-\Phi(p^\dagger)\right)>0,$$

where $\Phi(.)=\frac{4ab+2ad+2bd\sum_{j\neq i}^{n}\tilde{c_j}+d^2(N-1)\hat{c}(.)}{(2b+d)(2b-d(N-1))}$.

With a per unit subsidy of s^*, $q^*(p^\dagger,p^\dagger)=0$. All entrant types $c_i<p^\dagger$ can enter and incumbents can hold the corresponding belief $c^*=p^\dagger$ credibly. Beliefs and strategies are sequentially rational, thus constituting a PBE.

Expected industry output. Now, we consider the impact of facilitating entry on aggregate output.

The expected aggregate output is:

$$Q^*(c^*)=F_i(c^*)Q_N^*(c^*)+\big(1-F_i(c^*)\big)Q_{N-1}^*,$$

where

$$Q_N^*(c^*)=b\left[\frac{N(2ab+ad)+\left[d^2(N-1)-2b^2+2bd(N-3)\right]\left(\hat{c}(c^*)+\sum_{j\neq i}\tilde{c}_j\right)}{(2b+d)\big(2b-d(N-1)\big)}\right]$$

and

$$Q_{N-1}^*=b\left[\frac{(N-1)(2ab+ad)+\left[d^2(N-2)-2b^2+2bd(N-5)\right]\sum_{j\neq i}\tilde{c}_j}{(2b+d)\big(2b-d(N-2)\big)}\right],$$

are the expected values of industry output, obtained by aggregating the values of quantities corresponding to the equilibrium prices in Lemma 1. Similar to the entrant's output, we derive the first-order condition for aggregate output maximizing cutoff and show that:

Proposition 2. *Maximizing expected industry output requires facilitating entry above the level required to maximize expected entrant's output. The cost cutoff which maximizes $Q^*(c^*)$ is*

$$c^*=p^\Omega=\frac{2ab+ad+bd\sum_{j\neq i}\tilde{c}_j}{(2b-d(N-2))(b-d(N-1))}>p^\dagger>c_0^*.$$

This result is evident from the observation that raising the cutoff has a positive effect not only on the entrant's expected output, but through the incumbents output and pricing strategy. Thus, to the extent the gains are not exhausted for the incumbents, the cost cutoff has to be raised above $p^\dagger$ to maximize the industry output. This can be implemented using an appropriate per unit subsidy

$$s^{**} = \frac{(2b+d)(2b-d(N-1))}{2b(2b-d(N-1))+2d(b-d(N-1))}\left(p^\Omega - \Phi(p^\Omega)\right) > 0.$$

3.2 Welfare standards

In this section, we analyse the effect of changing the equilibrium cutoff on total welfare. To characterize total welfare, we first invert our direct demand function. Recall,

$$q_i = a - bp_i + d\sum_{j\neq i} p_j.$$

Upon inverting, we can express the indirect demand function as

$$p_i = \frac{a}{b-d(N-1)} - \frac{b-d(N-2)}{(b-d(N-1))(b+d)}q_i - \frac{d}{(b-d(N-1))(b+d)}\sum_{j\neq i} q_j.$$

Here, we strengthen Assumption 1 by increasing the required degree of product differentiation, such that $b > d(N-1)$. We can integrate and express the utility of a representative consumer as

$$U(.) = \frac{a}{b-d(N-1)}\sum_{h=1}^{N} q_h - \frac{1}{2}\frac{b-d(N-2)}{(b-d(N-1))(b+d)}\sum_{h=1}^{N} {q_h}^2 - \frac{1}{2}\frac{d}{(b-d(N-1))(b+d)}\sum_{\substack{j,h=1\\ j\neq h}}^{N} q_h q_j - \sum_{h=1}^{N} p_h q_h.$$

Differentiating the above with respect to outputs, we can recover our inverse demand function. Now, welfare is given by the sum of consumer surplus and producer surplus as follows:

$$\begin{aligned}
W &= CS + PS \\
&= U(.) + \sum_{h=1}^{N} p_h q_h - \sum_{h=1}^{N} c_h q_h \\
&= \frac{a}{b-d(N-1)}\sum_{h=1}^{N} q_h - \frac{1}{2}\frac{b-d(N-2)}{(b-d(N-1))(b+d)}\sum_{h=1}^{N} {q_h}^2 - \frac{1}{2}\frac{d}{(b-d(N-1))(b+d)}\sum_{\substack{j,h=1\\ j\neq h}}^{N} q_h q_j - \sum_{h=1}^{N} c_h q_h.
\end{aligned}$$

Upon taking expectations, we will have variance terms on account of the squared outputs in welfare. From the quantities corresponding to the equilibrium prices in (8)-(10), the output variance of each incumbent j is

$$V[q_j^*(.)] = \frac{b^2}{4}V[c_j],$$

and the conditional variance of entrant's output upon entry is

$$V[q_i^*(.)|c_i < c^*] = \frac{b^2}{4}V[c_i|c_i < c^*].$$

Total welfare is increasing in the first term, but decreases in output variance and production costs. We do not include subsidies as they can be considered a transfer payment from government to producers, effectively cancelling out from a social welfare perspective. Assuming for tractability that incumbents are *ex ente* symmetric (i.e. iid production costs), and normalizing $a = b = 1$, such that d is the inverse measure of product differentiation, we can show:

Proposition 3. *Total welfare is maximized at a cutoff $c_w^* > p^\dagger$. Hence, maximizing total welfare requires facilitating entry more than needed to maximize entrant's expected output.*

4 Generalization to all entrants game

In this section, we generalize the analysis to the case where all $N \geq 2$ firms are potential entrants. For tractability, assume marginal costs are iid over the support $[0, \bar{c}]$, with cdf $F(.)$, density $f(.)$ and mean $\tilde{c}$. We ensure that, conditional on entry, firms always sell a strictly positive output by binding the cost support from above as follows:

$$\bar{c} \leq \check{c} = \frac{2a + d(N-1)\tilde{c}}{2b - d(N-1)}.$$

With this, when entry is unrestricted (i.e. $k = 0$), all N firms enter for all $c_i \in [0, \bar{c}]$.

Expected aggregate output. Each firm enters with probability $F(c^*)$ in a symmetric equilibrium. Denote by

$$Q_n^*(c^*) = \frac{bn\left[a - (b - d(n-1))\hat{c}(c^*)\right]}{2b - d(n-1)}$$

the aggregate output when n firms enter. The expected aggregate output is

$$\mathbb{E}[Q_n^*(c^*)] = \sum_{n=0}^{N} \binom{N}{n} F(c^*)^n \left[1 - F(c^*)\right]^{N-n} Q_n^*(c^*).$$

Differentiating with respect to c^* and rearranging, we can show that

$$\lim_{c^* \to \bar{c}} \frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*} > 0 \iff \bar{c} < \frac{2ab + ad + bd(N-1)\tilde{c}}{(2b - d(N-2))(b - d(N-1))} = p^{\Omega},$$

which is always true. Hence, we can write:

Proposition 4. *It is optimal to let all firms enter the market irrespective of their marginal costs.*

5 Conclusion

This paper extends the analysis of Bisceglia et al. (2024) to a Bayes–Bertrand setting with differentiated products. We show that when firms compete in prices and possess private information about their marginal costs, output and welfare maximization may require facilitating the entry of relatively inefficient firms that would otherwise be excluded.

The mechanism is driven by the informational structure of the model. In a Bayesian Bertrand environment, firms condition their pricing decisions on the expected costs of their rivals rather than on realized costs. Because prices are strategic complements, facilitating entry raises incumbents' expectations about the cost of potential entrants. Anticipating a less efficient entrant, incumbents set higher prices, which in turn allows entrants to charge higher prices and expand output. As a result, facilitating entry increases entrant output, aggregate output, and total welfare.

A Proofs

Proof of Lemma 1

To arrive at the equilibrium cutoff c^*, we look at the entry condition at the margin:

$$\pi_i^*(c^*, \mathbf{1}) = k$$
$$\implies b(p_i^*(c^*, \mathbf{1}) - c^*)^2 = k$$
$$\implies p_i^*(c^*, \mathbf{1}) - c^* = \sqrt{\frac{k}{b}}$$

Now, we know that $b(p_i^*(c^*, \mathbf{1}) - c^*) = q_i^*(c_i, \mathbf{1})$. Upon substituting this and the exression from (7):

$$\frac{1}{2}\left[\frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n} \tilde{c}_j + d^2(N-1)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))} - c^*\right] = \sqrt{\frac{k}{b}} \quad \square$$

For the unrestricted entry cutoff c_0^*, we simply set $k = 0$ to get:

$$c_0^* = \frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n} \tilde{c}_j + d^2(N-1)\hat{c}(c_0^*)}{(2b+d)(2b-d(N-1))}.$$

Proof of Lemma 2

From (6):

$$\hat{c}(c^*) = E[c|c < c^*]$$
$$= \frac{\int_0^{c^*} c f_i(c)dc}{F_i(c^*)}$$
$$= \frac{N(c^*)}{D(c^*)}$$

Now, differentiating with respect to c^*:

$$\frac{\partial \hat{c}(c^*)}{\partial c^*} = \frac{N'(c^*)D(c^*) - N(c^*)D'(c^*)}{(D(c^*))^2}$$
$$= \frac{c^* f_i(c^*)F_i(c^*) - f_i(c^*)N(c^*)}{(F_i(c^*))^2}$$
$$= \frac{f_i(c^*)}{F_i(c^*)}\left[c^* - \hat{c}(c^*)\right] > 0 \quad \square$$

Note that $c^* > \hat{c}(c^*)$ from (5).

Proof of Proposition 1

Differentiating $q_i^*(c^*)$ with respect to c^* gives:

$$\frac{\partial q_i^*(c^*)}{\partial c^*} = f_i(c^*)\, b \left[\frac{2ab + ad + bd\sum_{j\neq i}^{n} \tilde{c}_j + (d^2(N-1) - 2b^2 + bd(N-1) - bd)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))}\right]$$
$$+ F_i(c^*)\, b \left[\frac{d^2(N-1) - 2b^2 + bd(N-1) - bd)}{(2b+d)(2b-d(N-1)}\right] \frac{\partial \hat{c}(c^*)}{\partial c^*} = 0$$

Substituting (10) in the above expression yields:

$$\frac{\partial q_i^*(c^*)}{\partial c^*} = f_i(c^*)\, b \left[\frac{2ab + ad + bd\sum_{j\neq i}^{n} \tilde{c}_j}{(2b+d)(2b-d(N-1))} + \frac{(d^2(N-1) - 2b^2 + bd(N-1) - bd)}{(2b+d)(2b-d(N-1))} c^*\right] = 0$$

$$\Longrightarrow\ c^* = \frac{2ab + ad + bd\sum_{j\neq i}^{n} \tilde{c}_j}{2b^2 + bd - d^2(N-1) - bd(N-1)} = p^\dagger \quad \square$$

Now, we shall look at the sign of $p^\dagger$. Clearly, the numerator is strictly positive. The sign of $p^\dagger$ then depends completely upon the denominator $2b^2 + bd - d^2(N-1) - bd(N-1)$. Observe that the denominator can be written as

$$b(2b - d(N-1)) + d(b - d(N-1)) > 0 \quad \text{if} \quad 2b > d(N-1)$$

Therefore, $p^\dagger$ is positive.

The second order condition can be easily checked as follows:

$$\left.\frac{\partial^2 q_i^*(c^*)}{\partial c^{*2}}\right|_{c^*=p^\dagger} = -\frac{2b^2 + bd - d^2(N-1) - bd(N-1)}{(2b+d)(2b-d(N-1))} f_i(p^\dagger) < 0.$$

To show that $p^\dagger > c_0^*$, note that from (9):

$$\left.\frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n} \tilde{c}_j + d^2(N-1)\hat{c}(c^*)}{(2b+d)(2b-d(N-1))} - c^*\right|_{c^*=c_0^*} = 0$$

Now, if we can show that the left-hand side is decreasing in c^*, then $p^\dagger > c_0^*$ if and only if

$$\frac{4ab + 2ad + 2bd\sum_{j\neq i}^{n} \tilde{c}_j + d^2(N-1)\hat{c}(p^\dagger)}{(2b+d)(2b-d(N-1))} - p^\dagger < 0$$
$$\Longleftrightarrow\ \hat{c}(p^\dagger) < p^\dagger$$

which is always true.

To show that the left-hand side is decreasing in c^*:

$$\frac{\partial LHS}{\partial c^*} = \frac{d^2(N-1)}{(2b+d)(2b-d(N-1))}\frac{\partial \hat{c}(c^*)}{\partial c^*} - 1 < 0$$
$$\Longleftrightarrow\ \frac{\partial \hat{c}(c^*)}{\partial c^*} < \frac{(2b+d)(2b-d(N-1))}{d^2(N-1)}$$

which is true by Assumption 2. $\square$

Proof of Proposition 2

Differentiating $Q^*(c^*)$ with respect to c^* gives:

$$\frac{\partial Q^*(c^*)}{\partial c^*} = f_i(c^*)\,\frac{b(b-d(N-1))}{2b-d(N-1)}\left[\frac{2ab+ad+bd\sum_{j\neq i}^{n}\tilde{c}_j}{(2b-d(N-2))(b-d(N-1))} - \hat{c}(c^*)\right]$$
$$+\,F_i(c^*)\,b\left[\frac{d^2(N-1)-2b^2+2bd(N-1)-bd)}{(2b+d)(2b-d(N-1)}\right]\frac{\partial \hat{c}(c^*)}{\partial c^*} = 0$$

Substituting (10) in the above expression yields:

$$\frac{\partial Q^*(c^*)}{\partial c^*} = f_i(c^*)\,\frac{b(b-d(N-1))}{2b-d(N-1)}\left[\frac{2ab+ad+bd\sum_{j\neq i}^{n}\tilde{c}_j}{(2b-d(N-2))(b-d(N-1))} - c^*\right] = 0$$

$$\implies c^* = \frac{2ab+ad+bd\sum_{j\neq i}^{n}\tilde{c}_j}{(2b-d(N-2))(b-d(N-1))} = p^{\Omega} \quad \square$$

Now, to show that

$$p^{\Omega} > p^{\dagger}$$
$$\iff 2b^2+bd-d^2(N-1)-bd(N-1) > (2b-d(N-2))(b-d(N-1))$$
$$\iff 2b > d(N-1)$$

which is true by Assumption 1. $\square$

Proof of Proposition 3

To prove that welfare is maximized at a cutoff $c^*_w > p^{\dagger}$, we can show that [3]

$$\left.\frac{\partial \mathbb{E}[W(\cdot)]}{\partial c^*}\right|_{c^*=p^{\dagger}} > 0 \quad \square$$

Proof of Proposition 4

Differentiating expected aggregate output with respect to c^*,

$$\frac{\partial \mathbb{E}[Q^*_n(c^*)]}{\partial c^*} = \sum_{n=0}^{N}\binom{N}{n}\Big[nF(c^*)^{n-1}f(c^*)[1-F(c^*)]^{N-n}Q^*_n(c^*)$$
$$-\,F(c^*)^n(N-n)[1-F(c^*)]^{N-n-1}f(c^*)Q^*_n(c^*)$$
$$+\,F(c^*)^n[1-F(c^*)]^{N-n}\frac{\partial Q^*_n(c^*)}{\partial c^*}\Big].$$

Facilitating entry increases expected aggregate output if and only if $\left.\frac{\partial \mathbb{E}[Q^*_n(c^*)]}{\partial c^*}\right|_{c^*\to\bar{c}} > 0$. Note that, as $F(c^*)\to 1$ for $c^*\to\bar{c}$, all the terms in the above derivative vanish except those for $n\in\{N-1, N\}$.

[3] The expressions of the welfare objective are very cumbersome, and the first order conditions cannot be solved analytically. Mathematica files available upon request.

We then have

$$\begin{aligned}\left.\frac{\partial \mathbb{E}[Q_n^*(c^*)]}{\partial c^*}\right|_{c^* \to \bar{c}} &= \frac{-b(N-1)}{2b-d(N-2)} \binom{N}{N-1} \left[a-(b-d(N-2))\tilde{c}\right] \\ &\quad + \frac{bN}{2b-d(N-1)} \binom{N}{N} \left[N\big(a-(b-d(N-2))\tilde{c}\big) - (b-d(N-1))(\bar{c}-\tilde{c})\right] \\ &> 0 \\ \iff \quad p^\Omega &> \bar{c}\end{aligned}$$

which is always true because

$$\bar{c} \leq \check{c} = \frac{2a+d(N-1)\tilde{c}}{2b-d(N-1)} < \frac{2ab+ad+bd(N-1)\tilde{c}}{(2b-d(N-2))(b-d(N-1))} = p^\Omega \quad \square$$

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