A Model of Airline Pricing: Capacity Constraints and Deadlines

Shubhro Sarkar*†

Abstract

We study monopolistic pricing, with a capacity constraint, of a good that loses its value after three periods. In each period a continuum of buyers, each of whom might be one of two types, enter. Each of the buyers chooses either to make a purchase as soon as they enter, or to wait for a lower price. The price path is found to be strictly non-decreasing, u-shaped or horizontal for different proportions of buyers with a higher willingness to pay. Any strategy involving 'final sales' is non-optimal. The predictions are empirically tested.

Keywords: Dynamic pricing, capacity constraints, time-sensitive goods, subgame perfection.
JEL codes: L10, L93, C72.

1 Introduction

It is a well-known fact that passengers on the same flight, traveling in the same class, often end up paying different prices for their tickets. This is because the prices of such tickets vary over time, often within the span of a few hours. While buyers have the option of purchasing tickets months prior to the date of departure, casual observation suggests that the prices offered by an airline are high

*Indira Gandhi Institute of Development Research, Mumbai, 400065, India. Email: shubhro@igidr.ac.in

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if a purchase is attempted too early, drop after a period of time and then prior to departure they rise again. An empirical study byb Stavins (2001) indicates that five weeks prior to departure, prices start rising. Instead of monotonically reducing prices, selling every available seat and waiting for takeoff, the airline instead, chooses to save a certain number of seats for future buyers, who would be willing to pay a high price for the same seats. This shows, that in order to solve for the optimal price path of such goods, we need a model with a finite time horizon, where one or many sellers while facing a capacity constraint, offer(s) a finite measure of units for sale. In each period, a continuum of buyers, each of whom might be one of two types, enters the market. The seller chooses, without precommitment, price and measure of units to offer in each period, while each of the buyers choose either to make a purchase as soon as they enter, or to wait for a lower price which might be made available in the future.

The operations research literature identifies airline ticket pricing as dynamic pricing (also known as yield management), where the product ceases to exist at a certain point in time and capacity can only be added at a very high marginal cost. The product being discussed here is non-durable, non-storable and cannot be resold. We could consider an airline ticket to be a futures contract on a service to be provided by the airline in the future. As the airline attempts to sell tickets over time, it is in effect "signing" contracts with different customers on different terms. As the seller is unable to precommit to the terms of the contract in the future and is in effect competing against future versions of himself (herself), he (she) faces the same intertemporal and time-consistency problems as a durable-goods monopolist. Other examples of such products include hotel rooms, generated electricity or other "sell before"goods where transactions occur through a futures contract (McAfee & Velde). Given the similarities in the problems facing an agent signing multiple futures contracts (airline) and a durable goods monopolist, we can refer to the vast literature on time-consistency issues in a durable-goods monopoly.

In this paper, we use a model which extends that of a durable-goods monopoly model by Conlisk, Gerstner and Sobel (CGS, 1984). In their infinite time horizon model, a new cohort of consumers enters the market in each period. The consumers in each group differ amongst themselves in terms of the valuation for the good. Some of these buyers choose to make a purchase in the same period, while others decide to wait for a lower price. Usually the single seller, who does not face any capacity constraint, prefers to sell the product at a price just low enough to sell immediately to consumers with a high willingness to pay, as long as revenue earned from selling to "high" type buyers exceeds revenue earned from selling to “low” type buyers. However, as sufficient number of consumers with a lower willingness to pay accumulate in the market, the seller holds a ‘sale’ by dropping the price low enough, so that buyers with lower willingness to pay can buy the product. This leads to an equilibrium where periodic ‘sales’ are held and the corresponding price path is cyclic. We extend this model, by introducing a capacity constraint for the single seller and by solving for the equilibrium for a finite time horizon.

The main predictions of the theoretical model are as follows. First, a sufficiently patient seller never offers any ‘sale’ in the last period. This is because the seller chooses to reserve some units for sale in the last period and offer them at high prices to high valuation buyers who enter the market in that period. Second, the measure of units offered for sale in any period where the seller chooses to offer the good to both types of buyers is a decreasing function of the proportion of high type buyers in the market. Third, the shape of the price path is horizontal, u-shaped or strictly non-decreasing for various ranges of parameter values. For example, for routes with the highest proportion of high type buyers, sellers had no incentive to offer a sale and the price path is horizontal. Routes with lower proportions of high type buyers have price paths which are strictly non-decreasing or u-shaped.

We collected data on prices over 15 weeks for 30 routes in the US. While the first prediction was found to be empirically valid, we find little evidence to support the hypothesis that the price path should be horizontal for routes with the highest proportion of buyers with a higher willingness to pay. We classified the routes into low, medium and high proportion of high type buyers and find that prices increase as the date of departure grows closer for all three types of routes. The rate of increase was highest for routes with the highest fraction of high type buyers. We did find some evidence for a u-shaped price path for routes with low or medium proportion of high type buyers when we looked at the last 10 weeks of observations.

2 Review of Literature

As mentioned in the previous section, even though airline tickets are not durable, the intertemporal problems facing a seller of airline tickets are identical to those facing a durable goods monopolist. We thus begin the review of literature section by referring to the literature on durable goods monopoly. The problem of intertemporal price discrimination as faced by a durable-goods monopolist has been the focus of several papers over the years. In his seminal paper, Coase (1972) conjectured that a durable-goods monopolist would be unable to exert any monopoly power. This is because rational buyers would anticipate correctly that in the absence of precommitment to future prices, the monopolist would reduce prices in an attempt to cater to residual demand and would refuse to buy the product as long as prices remained above the competitive level.

There are two assumptions, which are crucial to our model. The first is that the seller faces a capacity constraint, while the second is the constant influx of new buyers. It has been found that the Coase conjecture fails to hold under these assumptions. McAfee and Wiseman (2003) show that capacity costs of arbitrarily small degree can eliminate the zero profit conclusion. Capacity costs borne by the seller serve as a strong commitment device, as the choice of capacity enables the seller to slow the sales, reduce the fall in prices and thus permits the seller to set initial prices above marginal costs. Papers by Sobel (1984), Conlisk, Gerstner and Sobel (1984) show that the equilibrium in a model with a continual influx of new buyers involves price cycles where each seller produces a homogeneous good and sells it to consumers with different willingness to pay. A cyclic price path is also obtained in a paper by Narasimhan (1989), who uses a framework similar to that of CGS but assumes that the entry of new consumers is governed by a diffusion process. In his model, the number of buyers who enter the market in each period is a function of cumulative sales and is time variant. Unlike Conlisk et al. the market size in his model is fixed, such that after some time saturation effects set in.

An alternate outlook is presented in papers by Brumelle and McGill (1993) and Wollmer (1992), who solve for an optimum airline seat booking policy, where lower fare class customers book tickets before higher fare class passengers. In these papers, airlines solve for a critical number of seats in each fare class, which are reserved for potential future passengers who are willing to pay a higher price. Booking requests for a particular fare class are accepted if and only if the number of empty seats is strictly greater than its critical level and rejected otherwise. Wollmer shows that this critical value is a decreasing function of the fare price and is equal to zero for the highest fare (class). However, these papers lack the flavor of durable goods, as buyers do not have the option of staying in the market to wait for a lower price, while sellers do not compete with future incarnations of themselves.

Stavins (2001) addresses the issue of how airline prices move over time in a paper in which she examines how price discrimination changes with market concentration in the airline market. Price discrimination is found to increase as the markets become more competitive. The data set included fares offered 35 days prior to departure, followed by 21 days prior to departure, 14 days prior to departure and finally 2 days prior to departure. The data thus allowed for examination of how prices change as the departure date drew closer. From the OLS regression it was discovered that cheaper fares disappear, leaving only more expensive tickets for sale.

McAfee and Velde (2004) provide an extensive survey of yield management research in operations research journals and then test the predictions of these models with airline pricing data collected from 1,260 flights. They test the following five propositions. First, prices fall as the date of departure approaches. Second, prices rise initially. Third, competition reduces the variance in prices. Fourth, prices change as the number of empty seats remaining change and finally fifth, prices of flights leaving from substitute airports or departing at substitute times are correlated. They find that prices increased $50 in the week before takeoff on top of a rise of $28.20 the previous week. Thus the first proposition was empirically false and theories which assume that customers arriving in the market at different points in time are identical are invalid. Overall, there was scant empirical evidence in favor of the major theoretical predictions. [1] However, the routes considered by them had multiple airlines serving them, such that their results are inapplicable for models with a single seller.

Etzioni et al (2003) devise an algorithm called Hamlet, which when trained on a data set comprising of over 12,000 observations over a 41 day period, was able to generate a predictive model which enabled 607 simulated passengers an average savings of 27%. Flights were found to have discernible price tiers and the number of such tiers varied from two to four, depending on the airline and the particular flight. They find that pricing policies tend to be similar for airlines belonging to the same category and that the prices fluctuate more and are more expensive for bigger airlines. Finally, they observe that prices increase two weeks prior to departure which corroborates the empirical finding of Stavins.

The main contribution of this paper is to provide insights into the relationship between the proportion of buyers with a higher willingness to pay and the corresponding shape of the price path, from a model which incorporates some essential features of an airline pricing problem. The paper proceeds as follows. In the following section a basic three-period model is developed and strategies

[1] They do not specify as to whether the second proposition was found to be empirically valid.

for both players outlined. Section 4 identifies possible candidates for subgame perfect outcomes and describes conditions under which we get the different price paths. Section 5 describes the data, 6 the empirical model while section 7 presents the results. Section 8 contains the conclusions.

3 Model

Setting. Time is discrete. We can consider a finite horizon model of T periods. The good has a lifetime of T periods, after which it is assumed to be lost forever. In order to consider a simple version, we assume that $T = 3$.
Supply side. There is a single seller of the product. The monopolist faces constant marginal cost, assumed without loss of generality to be zero. The total measure of units of the product (seats) available to the monopolist is 3. The seller chooses to offer a continuum of units of measure $q_i \in [0,3]$ for $i = 1,2,3$ in period i. The seller also chooses price p_i for period i, so as to maximize sum of discounted revenue earned, calculated at discount factor ρ, with $0 < \rho < 1$. The monopolist cannot rent the product; at any given date, the monopolist cannot make binding commitments about future prices and measure of units to be offered for sale.
Demand side. A continuum of buyers of measure 2 enter the market in each period, with each buyer having unit demand. Buyers in each cohort can be one of two types. A continuum of buyers of measure 2α (with $0 < \alpha < 1$) enter the market in each period and have valuation for the product given by V_1, while a continuum of buyers of measure $2(1-\alpha)$ enter the market in each period and value the good at V_2, where $V_1 > V_2 > 0$. Buyers with valuation V_1 are said to be of 'high' type, while buyers with valuation at V_2 are said to be of 'low' type. We assume that the majority of buyers entering the market in each period are of low type and hence $\alpha \in (0, 1/2)$.

Buyers are rational. Each buyer on entering the market decides either to purchase the product in the current period or to wait for a lower price, except for buyers in the last period, who either decide to buy or not to buy the product in the last period. In the event that the buyer is indifferent between buying in the current period and waiting (or not to buy), the buyer is assumed to make the purchase immediately. Buyers assume that their own decision as to when to buy the product has no bearing on other buyer's decision as to whether and when to buy the same product. This is a consequence of the assumption that we have a continuum of buyers in the market. The probability that the buyer will get the product in period i is given by Φ_i which is determined endogenously. Once a consumer buys the product, he or she leaves the market forever. A consumer who has not bought the product stays in the market till period 3, regardless of when he or she first entered the market. Finally, no resales are allowed. All consumers are price takers, and they have no bargaining power. This, once again, is a consequence of the assumption that we have a continuum of buyers in the market.

Timing of events. At the beginning of period 1, the seller announces the price for the first period, p_1 and the measure of units available for purchase, q_1. A continuum of buyers of measure 2 enter the market in the first period, of which buyers of measure 2α are of 'high' type and buyers of measure $2(1-\alpha)$ are of 'low' type. Each buyer decides whether to buy the product in the first period, or to wait for a lower price which might be made available in the future. Based on p_1 and q_1, the seller knows the measure of units that were actually sold in the first period. At the beginning of the second period, the seller announces price for period 2, p_2 and the measure of units available for sale in the second period, q_2. A new cohort of buyers (of measure 2) enter the market in the second period. These buyers along with the buyers who decided not to buy the product in period 1 and hence chose to remain in the market then constitute the total measure of buyers in the market in the second period. Each of these buyers in turn decide either to purchase the product at price p_2 or to wait for a lower price in period 3. A similar sequence of events follow in period 3, except for the fact that buyers of both types in period 3, choose either to purchase or not to purchase the good in the last period.

We assume that the type of each buyer is *publicly observable*, such that we have a *complete information* model. We further assume that, even though the seller knows the type of each and every 'active' buyer in the market at any point of time, he or she is unable to price discriminate and must charge (or announce) a single price in every period.[2] We solve for the *subgame perfect outcomes* of the game described above for various combinations of parameter values. Conversely, we could have assumed that the buyers' types are not observable. In that case, Perfect Bayesian equilibrium would have been the appropriate equilibrium concept, where we would have to explicitly specify how

[2]By 'active' buyers we mean buyers who have chosen not to purchase the product in previous periods and have instead chosen to remain in the market for lower prices. 'Active' buyers in a particular period also include buyers who entered the market in the same period and are about to decide either to purchase the product or to wait for a lower price in periods 1 and 2, and either to purchase or not to purchase the product in period 3.

agents form beliefs for information sets on and off the equilibrium path.

Notation. The following notation is introduced in order to describe the total measure of 'high' and 'low' type buyers in the market at each point of time, as well as the measure of units left with the seller at the beginning of each period:

$b_i^H =$ Total measure of 'high' type buyers in the market including ones entering the market in period i.

$b_i^L =$ Total measure of 'low' type buyers in the market including ones entering the market in period i.

$s_i =$ Measure of units left with the seller at the beginning of period which is a function of $p_{i-1}, q_{i-1}, b_{i-1}^H, b_{i-1}^L$ and s_{i-1}, where p_{i-1}, q_{i-1} are control variables and $b_{i-1}^H, b_{i-1}^L, s_{i-1}$ are state variables for period $i-1$.

Transition Equations. If d_i denotes demand for the product in period i, while m_i represents measure of units actually sold in period i then $m_i = \min\{q_i, d_i\}$ where $d_i = \begin{cases} b_i^H \text{ if } \frac{V}{2} < p_i \le p_i^H \\ b_i^H + b_i^L \text{ if } p_i \le \frac{V}{2} \end{cases}$. Then, $s_{i+1} = s_i - m_i$ and

$$b_{i+1}^H = \begin{cases} 2\alpha + b_i^H \text{ if } p_i > p_i^H, \forall q_i \\ 2\alpha + (b_i^H - m_i) \text{ if } \frac{V}{2} < p_i \le p_i^H, q_i < b_i^H \text{ such that } m_i = q_i \\ 2\alpha \text{ if } \frac{V}{2} < p_i \le p_i^H, q_i \ge b_i^H \\ 2\alpha + b_i^H \left(1 - \dfrac{q_i}{b_i^H + b_i^L}\right) \text{ if } p_i \le \frac{V}{2}, q_i < b_i^H + b_i^L \\ 2\alpha \text{ if } p_i \le \frac{V}{2}, q_i \ge b_i^H + b_i^L \end{cases}$$

$$b_{i+1}^L = \begin{cases} 2(1-\alpha) \text{ if } p_i \le \frac{V}{2}, q_i \ge b_i^H + b_i^L \\ 2(1-\alpha) + b_i^L \left(1 - \dfrac{q_i}{b_i^H + b_i^L}\right) \text{ if } p_i \le \frac{V}{2}, q_i < b_i^H + b_i^L \\ 2(1-\alpha) + b_i^L \text{ if } p_i > \frac{V}{2}, \forall q_i \end{cases}$$

where p_i^H is the price in period i which makes 'high' type buyers indifferent between buying the product in period i and waiting for a lower price in period $i+1$.

A strategy for the monopolist specifies for each period, price and measure of units to be offered to the buyers as a function of the history of the game. A strategy for the buyer of each type on the other hand, specifies at each time and after each history (in which he or she has not previously purchased or in case he or she has just entered the market) whether to accept or to reject the monopolist's offered price.

Optimal Decision Rules. The optimal decision rule for the seller and for the buyers in period 3 is described as follows. The 'high' type buyer chooses according as

$$\text{In period 3, chosen action} = \begin{cases} \text{Buy in period 3 if } p_3 \leq V_1 \\ \text{Not buy otherwise} \end{cases} \tag{1}$$

The 'low' type buyer chooses according as,

$$\text{In period 3, chosen action} = \begin{cases} \text{Buy in period 3 if } p_3 \leq V_2 \\ \text{Not buy} \quad \text{otherwise} \end{cases} \tag{2}$$

The seller chooses p_3, q_3 in order to

$$\max_{p_3, q_3} p_3. \min\{q_3, d_3(p_3)\} \text{ subject to } q_3 \leq s_3 \tag{3}$$

where d_3 is defined as follows:

$$d_3 = \begin{cases} b_3^H \text{ if } V_2 < p_3 \leq V_1 \\ b_3^H + b_3^L \text{ if } p_3 \leq V_2 \end{cases}$$

In period 2, the 'high' type buyer chooses according as

$$\text{In period 2, chosen action} = \begin{cases} \text{Buy in period 2 if } p_2 \leq p_2^H \\ \text{Wait otherwise} \end{cases} \tag{4}$$

The 'low' type buyer chooses according as,

$$\text{In period 2, chosen action} = \begin{cases} \text{Buy in period 2 if } p_2 \leq V_2 \\ \text{Wait} \quad \text{otherwise} \end{cases} \tag{5}$$

where p_2^H is defined by the following equation

$$(V_1 - p_2^H) = \Phi_3(V_1 - p_3^*) \ with \ \Phi_3 = \begin{cases} \dfrac{q_3^*}{b_3^H} \text{ if } V_2 < p_3^* \leq V_1 \\ \dfrac{q_3^*}{b_3^H + b_3^L} \text{ if } p_3^* \leq V_2 \end{cases} \tag{6}$$

Here, we use a fixed-point argument. In period 2, the 'high' type buyers know b_2^H, b_2^L. For the time being they fix the p_2^H of all other 'high' type buyers and calculate the corresponding b_3^H and b_3^L. Given b_3^H, b_3^L these buyers can calculate the price and measure of units the seller will offer in period 3, p_3^* and q_3^*. Using p_3^*, q_3^* and equation (6) these buyers are able to recover a p_2^H which should be equal to the one originally assumed. p_2^H is thus the price the seller can charge in order to make the 'high' type buyers indifferent between buying in period 2 and waiting for a lower price in period 3. At the beginning of period 2, the seller announces p_2 and q_2 in order to

$$\max_{p_2,q_2} p_2.\min\{q_2, d_2(p_2)\} + \rho W(b_3^H, b_3^L, s_3) \text{ subject to } q_2 \le s_2 \tag{7}$$

where W is the continuation payoff earned by the seller in period 3 and d_2 is defined as

$$d_2 = \begin{cases} b_2^H \text{ if } V_2 < p_2 \le p_2^H \\ b_2^H + b_2^L \text{ if } p_2 \le V_2 \end{cases}$$

Finally, we describe the optimal decision rules for the seller and the buyers for period 1. The 'high' type buyer chooses according as

$$\text{In period 1, chosen action} = \begin{cases} \text{Buy in period 1 if } p_1 \le p_1^H \\ \text{Wait otherwise} \end{cases} \tag{8}$$

The 'low' type buyer chooses according as,

$$\text{In period 1, chosen action} = \begin{cases} \text{Buy in period 1 if } p_1 \le V_2 \\ \text{Wait} \quad \text{otherwise} \end{cases} \tag{9}$$

where p_1^H is defined by the following equation

$$(V_1 - p_1^H) = \Phi_2(V_1 - p_2^*) \; with \; \Phi_2 = \begin{cases} \dfrac{q_2^*}{b_2^H} \text{ if } V_2 < p_2^* \le p_2^{H*} \\ \dfrac{q_2^*}{b_2^H + b_2^L} \text{ if } p_2^* \le V_2 \end{cases} \tag{10}$$

At the beginning of period 1, the seller announces p_1 and q_1 in order to

$$\max_{p_1,q_1} p_1.\min\{q_1, d_1(p_1)\} + \rho W(b_2^H, b_2^L, s_2) \text{ subject to } q_1 \le 3 \tag{11}$$

where W is the continuation payoff earned by the seller in period 2 and d_1 is defined as

$$d_1 = \begin{cases} b_1^H \text{ if } V_2 < p_1 \le p_1^H \\ b_1^H + b_1^L \text{ if } p_1 \le V_2 \end{cases}$$

A *subgame perfect Nash Equilibrium (SPNE)* of this game will thus consist of a strategy profile, $\sigma = (S, B)$ where S specifies a strategy on the part of the seller which satisfies equations (3), (7) and (11) while B specifies strategies on the part of each buyer who decides either to buy or to wait for a lower price in periods 1 and 2, and either to buy or not to buy in period 3, which satisfies equations (1), (4), (8) for 'high' type buyers and equations (2), (5) and (9) for 'low' type buyers. The equilibrium is a *symmetric equilibrium* in the sense that in equilibrium all buyers of the same type, choose the same action in each period. With non-atomic buyers, unilateral deviations made by them affect neither the actions of other buyers or those of the monopolist. Thus, in order to check for subgame perfection, only unilateral deviations by the seller are considered. If the seller deviates, the players keep following the optimal rules described above from that point of time onwards. This means if a player discovers a history of the game at any stage, which is not consistent with the one expected in equilibrium, the player continues to follow his or her optimal decision rule from that time onwards.

As the seller announces p_i and q_i at the beginning of each period i, we define a *pricing policy* $(p_1, p_2, p_3, q_1, q_2, q_3)$ which describes the prices charged and the units offered for sale in each period. It is possible to derive eight such possible price paths, where in each period, the seller decides either to sell only to 'high' type buyers or to sell to both 'high' and 'low' types.

4 Candidates for Subgame Perfect Outcome

In this section, we examine the different possible pricing policies and the associated price paths from which the seller might choose, under different combinations of the parameters V_1, V_2, α and ρ. Since we are interested in decisions made by a patient seller, we further assume that $\rho \to 1$.

4.1 No 'sale' in any period and 'sale' in every period

The first pricing policy we consider is one where the seller chooses to sell only to 'high' type buyers in every period. The price charged in each period is V_1, while the measure of units offered for sale in each period is 2α.

The second pricing policy $(V_2, V_2, V_2, 2, 1, 0)$ also yields a horizontal price path, but this time, the seller chooses to sell to both 'high' and 'low' type buyers in every period.

4.2 'Sale' in the first period only

The seller could choose to hold a 'sale' in the first period only, where he or she offers to charge $p_1 = V_2$ and $p_2 = p_3 = V_1$. Given the price path and the measure of units offered in the first period, q_1, the measure of units to be offered in periods 2 and 3 should be $q_2 = b_2^H$ and $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

Lemma 1. *With $p_1 = V_2$ and $p_2 = p_3 = V_1$ if $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, then with $\rho \to 1$ the seller offers $q_1 = \frac{3-6\alpha}{1-\alpha}$, $q_2 = b_2^H = \frac{\alpha+2\alpha^2}{1-\alpha}$ and $q_3 = s_3 = b_3^H = 2\alpha$ and if $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, then he or she offers $q_1 = 2$, $q_2 = q_3 = 2\alpha$. If $\alpha > \frac{V_2}{V_1}$, then with $\rho \to 1$ the seller offers $q_1 = 0$, $q_2 = b_2^H = 4\alpha$ and $q_3 = s_3 = 2\alpha$.*

Proof. Available upon request. □

From the above lemma we find that with $\alpha \leq \frac{V_2}{V_1}$, the seller chooses to offer measure q_1 units in the first period in a way which ensures that $s_3 = b_3^H$, such that the seller will have no incentive to hold a 'sale' in the last period.

4.3 'Sale' in the first two periods

Another strategy for the seller could be to offer a measure of units at price V_2 in the first two periods, and to sell to 'high' valuation buyers in the last period. Given the price path and the measure of units offered in the first and second periods (q_1 and q_2 respectively), the seller should offer $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

Lemma 2. *With $p_1 = p_2 = V_2$ and $p_3 = V_1$ if $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, then with $\rho \to 1$ the seller offers $q_1 = \frac{3-6\alpha}{1-\alpha}$, $q_2 = 0$ and $q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$ and if $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$ then he or she offers $q_1 = 2$, $q_2 = \frac{1-4\alpha}{1-\alpha}$ and $q_3 = s_3 = b_3^H = \frac{3\alpha}{1-\alpha}$. If $\alpha > \frac{V_2}{V_1}$, then with $\rho \to 1$ the seller offers $q_1 = 0$, $q_2 = 0$ and $q_3 = 6\alpha$.*

Proof. Available upon request. □

Proposition 1. *If $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}, \rho \to 1$ then $(V_2, V_1, V_1, 2, 2\alpha, 2\alpha)$ cannot be a subgame perfect outcome.*

Proof. Available upon request. □

Proposition 2. *If* $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$, $\rho \to 1$ *then* $(V_2, V_2, V_1, \frac{3-6\alpha}{1-\alpha}, 0, \frac{3\alpha}{1-\alpha})$ *cannot be a subgame perfect outcome.*

Proof. Available upon request. □

Proposition 3. *If* $\alpha \leq \frac{V_2}{V_1}$, $\rho \to 1$ *then* $(V_2, V_2, V_2, 2, 1, 0)$ *cannot be subgame perfect.*

Proof. Available upon request. □

4.4 'Sale' in the first and last period

In case the seller chooses to hold a 'sale' in the first and last period, the corresponding price path is inverted u-shaped. The seller charges $p_1 = p_3 = V_2$ and $p_2 = p_2^H > V_2$. Given the price path and the measure of units offered for sale in period 1, q_1, the seller offers $q_2 = b_2^H$ and $q_3 = s_3$.

Lemma 3. *With* $\rho \to 1$, $p_1 = p_3 = V_2$ *and* $p_2 = p_2^H$ *the seller offers* $q_1 = 0$, $q_2 = b_2^H = 4\alpha$ *and* $q_3 = s_3 = 3 - 4\alpha$ $\forall$ $V_1 > V_2$.

Proof. Available upon request. □

In order to rule out profitable deviations in period 3, the seller must have no incentive to charge $p_3 = V_1$. The required condition to ensure this is $(3-4\alpha)V_2 > 2V_1\alpha \Rightarrow \alpha < \frac{3V_2}{2(V_1+2V_2)}$. Similarly, we also have to rule out profitable deviations in period 2, given the history of the game $p_1 = V_2$ and $q_1 = 0$.

Proposition 4. *With* $\rho \to 1$, $\left(V_2, V_1(1-\frac{3-4\alpha}{6-4\alpha}) + \frac{3-4\alpha}{6-4\alpha}V_2, V_2, 0, 4\alpha, 3-4\alpha\right)$ *is never subgame perfect.*

Proof. Available upon request. □

4.5 'Sale' in the second period only

For the strategy involving a 'sale' in the second period only, the price path generated is u-shaped. Since the seller holds a sale in the second period only, he or she charges $p_1 = p_1^H$ (to make 'high' type buyers indifferent between waiting and purchasing in period 1), $p_2 = V_2$ and $p_3 = V_1$. Given the price path and the measure of units offered in periods 1 and 2 as q_1 and q_2 respectively, the seller offers $q_3 = s_3$ (if $s_3 < b_3^H$) or b_3^H (if $b_3^H < s_3$).

Lemma 4. *With $p_1 = p_1^H, p_2 = V_2$ and $p_3 = V_1$ if $\alpha \leq \dfrac{2V_2}{V_1+V_2}$, then with $\rho \to 1$ the seller offers $q_1 = 2\alpha$, $q_2 = \dfrac{6\alpha^2-15\alpha+6}{2(1-\alpha)}$ and $q_3 = s_3 = b_3^H = \dfrac{5\alpha-2\alpha^2}{2(1-\alpha)}$. On the other hand, if $\alpha > \dfrac{2V_2}{V_1+V_2}$, then with $\rho \to 1$ the seller offers $q_1 = 2\alpha$, $q_2 = 0$ and $q_3 = 4\alpha$.*

Proof. Available upon request. □

As was the case with strategies involving 'sales' in the first period only or the first two periods, the seller offers measure q_1 and q_2 units in the first and second period in a way which ensures that if $\alpha \leq \dfrac{2V_2}{V_1+V_2}$ and $\rho \to 1$, $s_3 = b_3^H$ such that there is no incentive for the seller to hold a 'sale' in the last period. With $\alpha > \dfrac{2V_2}{V_1+V_2}$ the seller chooses to offer measure zero units for 'sale' in the second period.

Proposition 5. *If $\alpha \in \left[\dfrac{1}{4}, \dfrac{V_2}{V_1}\right]$, then with $\rho \to 1$ $\left[V_1(1-\dfrac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)})+\dfrac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \dfrac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \dfrac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ cannot be subgame perfect.*

Proof. Available upon request. □

4.6 'Sale' in the last two periods

For strategies involving 'sales' in the last two periods, the seller charges $p_1 = p_1^H, p_2 = p_3 = V_2$ and offers $q_1 = 2\alpha, q_2 = 3-2\alpha, q_3 = 0$. These are the only $q_i s$ which are time consistent.

Proposition 6. *If $\alpha \leq \dfrac{V_2}{V_1} < \dfrac{2V_2}{V_1+V_2}$ and $\rho \to 1$ then $\left[V_1\left(1-\dfrac{3-2\alpha}{4-2\alpha}\right)+\dfrac{3-2\alpha}{4-2\alpha}V_2,\right.$ $\left. V_2, V_2, 2\alpha, 3-2\alpha, 0\right]$ cannot be subgame perfect.*

Proof. Available upon request. □

4.7 'Sale' in the last period only

For 'sale' in the last period only, the seller sets $p_1 = p_1^H, p_2 = p_2^H$ to ensure that 'high' type buyers are indifferent between buying the good and waiting for the price V_2 in the last period. The seller offers $q_1 = q_2 = 2\alpha$ and $q_3 = s_3 = 3 - 4\alpha$. In this case, these are the only $q_i s$ which are time consistent. Given that $p_1 = p_1^H, p_2 = p_2^H$ and that $q_1 = q_2 = 2\alpha$, the seller will choose to offer $q_3 = s_3$.

Proposition 7. *If* $\alpha \le \frac{V_2}{V_1}$ *and* $\rho \to 1$ *then* $\left[V_1(1-\frac{3-4\alpha}{6-4\alpha})+\frac{3-4\alpha}{6-4\alpha}V_2, V_1(1-\frac{3-4\alpha}{6-4\alpha})+\frac{3-4\alpha}{6-4\alpha}V_2, V_2, 2\alpha, 2\alpha, 3-4\alpha\right]$ *cannot be subgame perfect.*

Proof. Available upon request. □

Proposition 8. *If* $\alpha \le \frac{V_2}{V_1}$ *and* $\rho \to 1$ *then* $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ *cannot be subgame perfect.*

Proof. Available upon request. □

So far, for a particular range of parameter values ($\alpha \le \frac{V_2}{V_1}, \rho \to 1$) we have shown which pricing policies cannot be subgame perfect. Now we turn our attention to policies which are subgame perfect for the same range of parameter values.

Proposition 9. *If* $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$ *and* $\rho \to 1$ *then* $(V_2, V_1, V_1, \frac{3-6\alpha}{1-\alpha}, \frac{\alpha+2\alpha^2}{1-\alpha}, 2\alpha)$ *is subgame perfect.*

Proof. Available upon request. □

Proposition 10. *If* $\alpha < \frac{1}{4}$ *(for* $V_1 \le 4V_2$*) and* $\alpha \le \frac{V_2}{V_1}$ *(for* $V_1 > 4V_2$*) and* $\frac{6\alpha^2-7\alpha+4}{\alpha(4\alpha+2)} < \frac{V_1}{V_2}$ *(with* $\rho \to 1$*) then the pricing policy involving a 'sale' in the second period only is subgame perfect.*

Proof. Available upon request. □

Proposition 11. *If* $\alpha < \frac{1}{4}$ *(for* $V_1 \le 4V_2$*) and* $\alpha \le \frac{V_2}{V_1}$ *(for* $V_1 > 4V_2$*) and* $\frac{6\alpha^2-7\alpha+4}{\alpha(4\alpha+2)} > \frac{V_1}{V_2}$ *(with* $\rho \to 1$*) then* $(V_2, V_2, V_1, 2, \frac{1-4\alpha}{1-\alpha}, \frac{3\alpha}{1-\alpha})$ *is subgame perfect.*

Proof. Available upon request. □

Proposition 12. *If* $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ *and* $\alpha \geq \frac{3V_2}{2(V_1+2V_2)}$, *then* $(V_2, V_1, V_1, 0, 4\alpha, 2\alpha)$ *is subgame perfect.*

Proof. Available upon request. □

Since the seller offers $q_1 = 0$, *any price* in period 1 can be supported as a subgame perfect outcome. Thus, if $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ and $\alpha \geq \frac{3V_2}{2(V_1+2V_2)}$, then $(p_1, V_1, V_1, 0, 4\alpha, 2\alpha)$ is subgame perfect.

Proposition 13. *If* $\alpha \in \left(\frac{V_2}{V_1}, \frac{2V_2}{V_1+V_2}\right]$, $\rho \to 1$ *and* $\alpha < \frac{3V_2}{2(V_1+2V_2)}$, *then* $\left[V_1(1-\frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}) + \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)(4-2\alpha)}V_2, V_2, V_1, 2\alpha, \frac{6\alpha^2-15\alpha+6}{2(1-\alpha)}, \frac{5\alpha-2\alpha^2}{2(1-\alpha)}\right]$ *is subgame perfect.*

Proof. Available upon request. □

Proposition 14. *If* $\alpha > \frac{2V_2}{V_1+V_2}$, $\rho \to 1$ *then* $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ *will be subgame perfect.*

Proof. Available upon request. □

4.8 Main Results and Intuition

Figure 1 shows the pricing policies which are subgame perfect for the different combinations of parameter values. For higher values of V_1 combined with high values for α, the seller chooses not to hold a 'sale' in any period, such that only 'high' valuation buyers get to purchase the good. For $\alpha \in \left(\frac{3V_2}{2(V_1+2V_2)}, \frac{2V_2}{V_1+V_2}\right]$ the seller chooses to offer $q_1 = 0, q_2 = 4\alpha, q_3 = 2\alpha$ and to charge any price p_1, $p_2 = p_3 = V_1$ which is equivalent to not offering to hold a 'sale' in any period. For the same range of parameter values, the seller cannot choose the pricing policy $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$ since there exists a profitable deviation for the seller by holding a 'sale' in the second period and to sell to 'high' valuation buyers in the last period. Had the seller been able to credibly precommit, he or she would have chosen the pricing policy $(V_1, V_1, V_1, 2\alpha, 2\alpha, 2\alpha)$. For lower values of

α, the seller chooses to hold a 'sale' in at least one period, and being patient, chooses to hold a 'sale' in the second period. The corresponding price path was u-shaped (shaded zone in figure 1). Finally, for the lowest values of V_1 and α, the seller chooses to have a 'sale' in two periods and thus charges price V_2 for the first two periods. [3]

[Place figure 1 here.]

Thus the main results of the theoretical model are as follows:

(1) Any strategy involving a 'sale' in the last period is not subgame perfect. In case the seller chooses to offer a 'sale' in one or both of the first two periods, the measure of units offered for 'sale' is chosen in way to ensure that the measure of units remaining with the seller at the beginning of the third period is equal to the measure of high type buyers who remain 'active' in the last period. Since $p_3 \in \{V_1, V_2\}$, revenue maximization in the last period requires the seller to cater only to high valuation buyers.

(2) The total measure of units offered in any period(s) in which a 'sale' is announced is a decreasing function of α. This means that as the proportion of high valuation buyers increases, the seller chooses to offer a smaller measure of units at price V_2. For example, in case the seller wants to offer $p_1 = p_2 = V_2$ and $p_3 = V_1$, then with $\alpha \in \left[\frac{1}{4}, \frac{V_2}{V_1}\right]$ and $\rho \to 1$, the seller offers $q_1 = \frac{3-6\alpha}{1-\alpha}, q_2 = 0$ and with $\alpha < \frac{1}{4} \leq \frac{V_2}{V_1}$, he (she) offers $q_1 = 2$ and $q_3 = \frac{1-4\alpha}{1-\alpha}$, such that $(q_1 + q_2)$ was a decreasing function of α. Similarly, for the case where the seller chooses to offer a 'sale' in the second period only, then with $\alpha \leq \frac{2V_2}{V_1+V_2}$ and $\rho \to 1$, the seller offers $q_2 = \frac{6\alpha^2 - 15\alpha + 6}{2(1-\alpha)}$ which is also decreasing in α.

(3) The price path is horizontal, u-shaped or strictly non-decreasing for various ranges of parameter values (see figure 1).

We collected data in order to test these predictions empirically. In the event the empirical results failed to match the theoretical predictions, we attempt to provide an intuitive explanation behind such a failure(s).

[3] Had we considered cases where ρ is much smaller than 1, we would have found combinations of V_1 and α which make $(V_2, V_2, V_2, 2, 1, 0)$ subgame perfect.

5 Data

While the theoretical model was highly stylized in the sense that it allowed us to capture certain features of the airline ticket pricing, it diverged from the airline ticket market in the following ways. First, we often observe last minute deals being offered by some airlines on online travel sites like Priceline. Such discounts are never made available directly from the airlines themselves. In this case, airlines wait till the last few days before the flight departs and offer these seats at a discount through some online travel agents, since selling them at a lower price is preferred to flying with empty seats. Our theoretical model did not allow for such strategies. Second, airline tickets usually come with various sorts of restrictions. Travel restrictions are placed on certain tickets being offered at cheaper rates to make them unattractive to price inelastic buyers (for example, Saturday-night stay-over). Consumers end up self selecting the type of ticket and its price which they find most attractive. However, the theoretical model constructed, did not allow for such purchase restrictions and had no quality differentiation for the product being sold.

We collected price data for *economy class* tickets for one-way, non-stop flights in the US. We thus consider tickets with the least number of restrictions. Further, these routes were hand-selected such that only a single carrier offered services on each of them. This was done to ensure that the airline was a monopoly on that particular route, since the theoretical predictions are valid only for a single seller framework and we were unsure of how the predictions would change for a multiple seller setup. Even though the theoretical model made predictions about the shape of the price path and the measure of units made available for sale in each period for different range of parameter values, we could only empirically test the predictions about the shape of the price path since the number of seats made available for sale by an airline over any period of time was not observable.

The data set consists of two main components. The first component contains airline pricing data on selected routes while the second describes the proportion of “high” type buyers on each of these routes.

5.1 Airline Price Data

We collected pricing data for 28 one-way, non-stop flights and for 2 two-way non-stop flights from Expedia and Orbitz. The data was collected twice a day, at 8AM and 8PM, for 14-15 weeks (except for one flight for which we have 11 weeks of observations), which led to a total of 6136 observations. A total of 14 airlines operated on these routes which consisted of 44 distinct cities, of which some were major players like American and Delta, while others were smaller carriers like Midwest Airlines and Frontier Airlines.

The routes and flights selected had the following features: (1) Each route had a single airline operating on it. (2) Routes with a single airline but with more than two flights operating on a single day were excluded. Routes which had two flights which departed within a few hours of each other were also omitted. The selected routes had a maximum of two flights operating on them on any given day and in the event there was more than one flight, the flights departed at least 3 hours apart.[4] Thus, the selected flights had little or no competition.

The selected routes, the carriers serving them and the dates of departure, are listed in table 7. All flights departed in early June, 2005. The flights to Kahului and Honolulu were two-way, both having return dates on June 17, 2005. We purposely chose these dates following an observation by Etzioni at al, that prices bounce around more for flights leaving around holidays than others.

5.2 Data on Proportion of "High" Type Buyers

We use the American Travel Survey (ATS, 1995) as the source for data on the proportion of buyers with a higher willingness to pay. The survey contains data at the state and metropolitan area (MA) levels and describes trip characteristics for both households and individuals. Given a MA, trip characteristics for an individual person are arranged in the following sequence. First, the survey reports "person trip characteristics" given the MA as destination and the different census divisions (CD) as origin. Second, it displays the same characteristics for the same metropolitan area as origin and the various CDs as destination. Next, taking the MA as destination, the survey presents trip characteristics for the most frequent state origins. These states are the ones with the 10 largest volumes of travel to that particular MA. Fourth, taking the MA as origin, the corresponding numbers are listed for the states which are the top 10 destinations. Finally, the same order is followed for the cities which are the most frequent origins and destinations for travel to and from that particular MA.

Trip characteristics amongst others included "main purpose of trip", which was further categorized into business, pleasure and others. Ideally, we would

[4]Of the 30 routes, only 5 had two flights operating on them on any given day.

want the percentage of travelers who traveled by plane for business purposes from one MA to another. However, these numbers were not available.

The survey also, did not report data for any of the routes on which both MAs were represented as origin and destination. Thus, while we had data for both San Francisco and Kansas City MAs individually, the proportion of business travelers traveling from San Francisco to Kansas City were unavailable. This is because on one hand San Francisco was not amongst the top 10 cities having the most travel volume going to Kansas City and on the other, Kansas City was not amongst the top 10 cities having the most travel volume coming from San Francisco.

While we could have avoided this problem by looking at routes like New York-Boston and San Francisco-Phoenix for which we would have the corresponding percentages of travelers traveling for business purposes, these routes had a number of airline carriers flying on them, which made these markets oligopolies instead of monopolies, and unsuitable for consideration. We chose to fix the destination city and looked at the percentages of travelers traveling for business purposes (includes all forms of transportation), from the CD to which the city of origin belonged. For example, for the flight from New Orleans to Boston, we fixed the destination city (Boston) and looked at the proportion of business travelers traveling from the West South Central CD to which Louisiana belongs. This meant that we could use only 19 of the original 30 routes selected for data collection. The proportions of "business" travelers traveling on the different routes are reported in table 8.

6 Empirical Model

Since the main purpose of this section is to test the theoretical predictions outlined in section 4, we set up a number of empirical models which when estimated, delineates the relationship between the proportion of "high" type buyers on a route and the slope of the corresponding price path. We begin by estimating a model which assumes that the prices on any route depend on the number of observations left for departure and the proportion of "high" type buyers on that route. We will refer to this as model 1.

$$P_{mt} = \delta_m + \beta_1(\alpha_m D_{mt}) + \gamma_1 D_{mt} + \varepsilon_{mt} \tag{12}$$

where P_{mt} is the price for route m at time t, δ_m is a route specific intercept term (dummy) which remains constant over time, α_m denotes the proportion of business travelers on route m and D_{mt} is the number of observations left for departure on route m at time t. If a route has 15 weeks of observations collected twice a day, the variable D_{mt} takes values from 210 to 1. Thus, as D_{mt} decreases, we move closer to departure. From equation (14) we get,

$$\frac{\partial P_{mt}}{\partial D_{mt}} = \beta_1 \alpha_m + \gamma_1 \tag{13}$$

which implies that if the coefficient β_1 is not significant, α_m has no effect on the slope of the price path.

Next, we construct a model where we categorize the routes into ones with 'high', 'medium' and 'low' proportion of business travelers and assign dummies to them as follows. Assuming that α_m represents the proportion of business travelers on route m, we define for that route $\alpha_m = \alpha^H = 1$ if $\alpha_m > 0.45$ and 0 otherwise, $\alpha_m = \alpha^M = 1$ if $0.25 \leq \alpha_m \leq 0.45$ and 0 otherwise and finally $\alpha_m = \alpha^L = 1$ if $\alpha_m < 0.25$ and 0 otherwise. Thus, in addition to the route specific dummy variables, we construct a model with dummies which equal 1 or 0 depending on whether the route contains 'high', 'medium' or 'low' proportion of business travelers. We will refer to this as model 2.

$$P_{mt} = \delta_m + \beta_1(\alpha^H D_{mt}) + \beta_2(\alpha^M D_{mt}) + \gamma_1 D_{mt} + \varepsilon_{mt} \tag{14}$$

The estimates from this model will give us some idea about the slope of the price path for the three categories of routes. However, in order to obtain the shape of the price path we need to check how prices change over time. In the next step, we construct another model where we introduce dummies for number of weeks before departure. While the theoretical model had three periods, it is not apparent how we should define periods in the empirical counterpart. The theoretical model assumes that the measure of "high" type buyers entering the market in each period remains the same over the three periods. Typically, travelers with a higher willingness to pay for tickets enter the market in larger numbers in the weeks just prior to departure than earlier on. Thus, we introduce the dummies for number of weeks prior to departure as follows: $D^1 = 1$ for one week before departure, 0 otherwise, $D^2 = 1$ for 1 to 3 weeks before departure and 0 otherwise and $D^3 = 1$ for rest and 0 otherwise. The corresponding model (model 3) assumes the following form.

$$\begin{gathered} P_{mt} = \delta_m + \beta_1(\alpha^L D^1) + \beta_2(\alpha^L D^2) + \beta_3(\alpha^L D^3) + \gamma_1(\alpha^M D^1) + \\ \gamma_2(\alpha^M D^2) + \gamma_3(\alpha^M D^3) + \theta_1(\alpha^H D^1) + \theta_2(\alpha^H D^2) + \theta_3(\alpha^H D^3) + \varepsilon_{mt} \end{gathered} \tag{15}$$

Finally, we perform a Chow Breakpoint test to confirm whether there were structural changes in the price path before and after pre-determined cutoff points. To do this, we proceed using the following steps.

Step (1) We split the data set into two parts, such that with $D_{mt} \leq c$ (c being the pre-determined break point), the data is said to belong to group 1 and with $D_{mt} > c$ data is said to belong to group 2.

Step (2) We take $c = 105$. We then create a dummy variable which takes value 1 for $D_{mt} \geq 105$ and 0 otherwise and create another dummy variable ($time\_dummy2$) which takes value 1 for $D_{mt} < 105$ and 0 otherwise.

Step (3) Get estimates for the coefficients of the following model.

$$\begin{aligned} P_{mt} = K + \delta_m + \beta_1(\alpha_m D_{mt}) + \gamma_1(D_{mt}) + \beta_2(\alpha_m D_{mt} \times time\_dummy2) \\ + \gamma_2(D_{mt} \times time\_dummy2) + \theta_2 time\_dummy2 + \varepsilon_{mt} \end{aligned} \tag{16}$$

Since theory predicts a horizontal price path for routes with the highest α, we run the above regression only for those routes with $\alpha^H = 1$ and test for $\beta_2 = 0$, $\gamma_2 = 0$ and $\theta_2 = 0$. If the null hypothesis cannot be rejected, then there is no structural change in the model before and after the breakpoint.

7 Results

Table 1 reports the descriptive statistics for the data sets for the following two cases. (1) Includes all 30 routes for which different criteria are used for the proportion of business travelers on the different routes. For example, for the Austin-Washington DC route, we used the proportion of business travelers who traveled from Texas (state as origin) to DC and for the Seattle-Tucson route, we used the proportion of business travelers who flew from Austin. (2) Considers only 19 of the 30 routes, for which we fix the destination city and use the proportion of travelers traveling for business purposes from the CD to which to city of origin belongs, to the destination city.

All the equations were estimated using OLS. Route dummies were used to take into account route-specific characteristics, which remain unchanged over time. Since the use of miscellaneous criteria for the proportion of business travelers is unintuitive, we ran all the regressions for the 19 routes using the criteria as described in the second case above. Table 2 contains the estimates of the coefficients for equation (14).

Since both the coefficients are negative and significant, we can conclude from equation (15) that the slope of the price path is *negative*. However, since

	No. of Obs	Mean	St. Devn	Min	Max
All 30 Routes					
Price	6136	331.98	225.63	86	2441
Proportion of Business Travelers		0.336	0.173	0.03	0.74
19 Routes					
Price	3842	308.18	172.14	86	816
Proportion of Business Travelers		0.375	0.187	0.03	0.74

Table 1: Descriptive Statistics

	Coef.	St. Error	t-stat	p-Value
$\alpha_m \times D_{mt}$	-0.323	0.089	-3.63	0.000
D_{mt}	-0.197	0.026	-7.63	0.000

Table 2: Regression Results for Model 1 for 19 Routes

an increase in D_{mt} signifies movement away from departure, the negative slope obtained implies that prices increase as we move closer to departure. This result corroborates earlier findings of Stavins (2001), McAfee and Velde (2004) and Etzioni et al (2003).

The coefficients of model 2 could be interpreted as follows. Each route can only have either high, medium or low proportion of travelers with a high valuation. The coefficient for D_{mt} represents the base case and denotes the slope of the price path for routes which have $\alpha_m = \alpha^L$ (second and third terms drop out). The sum of the coefficients of $\alpha^M \times D_{mt}$ and D_{mt} refers to the slope of the price path for routes with $\alpha_m = \alpha^M$, while the sum of the coefficients of $\alpha^H \times D_{mt}$ and D_{mt} represents the slope of the price path for routes with $\alpha_m = \alpha^H$.

Thus, the slopes of the price path for routes with low, medium and high proportion of travelers with a high valuation are $-0.277, -0.217$ and -0.492 respectively. Prices are found to increase most quickly in routes with the highest proportion of business travelers.

	Coef.	St. Error	t-stat	p-Value
$\alpha^H \times D_{mt}$	-.215	.056	-3.85	0.000
$\alpha^M \times D_{mt}$	.060	.033	1.81	0.071
D_{mt}	-.277	.021	-13.30	0.000

Table 3: Regression Results for Model 2 for 19 Routes

	Coef.	St. Error	t-stat	p-Value
$\alpha^L \times D^1$	163.56	4.613	35.46	0.000
$\alpha^L \times D^2$	118.733	1.702	69.75	0.000
$\alpha^L \times D^3$	108.909	1.154	94.39	0.000
$\alpha^M \times D^1$	200.877	8.482	23.68	0.000
$\alpha^M \times D^2$	145.935	2.944	49.58	0.000
$\alpha^M \times D^3$	132.333	1.908	69.35	0.000
$\alpha^H \times D^1$	267.456	16.272	16.44	0.000
$\alpha^H \times D^2$	156.654	5.159	30.36	0.000
$\alpha^H \times D^3$	128.0702	2.319	55.22	0.000

Table 4: Regression Results for Model 3 for 19 Routes

The coefficients of model 3 allows us to demonstrate the relationship between the *shape* of the price path and the corresponding α_m. The price path is found to be rising for all three categories of routes (table 4). Routes with $\alpha_m = \alpha^H$ shows the sharpest increase in prices. The theoretical prediction that the price path for routes with high α is horizontal is thus found to be empirically invalid.

Finally, we report the results for the Chow Breakpoint test. For routes with $\alpha = \alpha^H$, theory predicts that there will be no change in the slope or the intercept before and after the break point. This implies that all three coefficients β_2, γ_2 and θ_2 need to be not significant for equation (18). Table 5 reports the coefficients for the Chow Breakpoint test for different pre-determined cutoff

c	β_1	γ_1	β_2	γ_2	θ_2	$F\,stat$	$p\,value$
105	0.413	−0.549	0.167	−1.043	55.908	33.99	0.000
70	0.562	−0.544	0.882	−2.774	105.632	35.66	0.000
42	0.109	−0.219	−3.148	−2.547	152.719	31.77	0.000

Table 5: Coefficients for Chow Breakpoint Test for Routes with $\alpha = \alpha^H$

values (c). The F-statistic is based on the null hypothesis which involves the restrictions, $\beta_2 = 0, \gamma_2 = 0$ and $\theta_2 = 0$. The low p-values led us to conclude that the null hypothesis can be rejected for all three pre-determined cutoff points and that there is structural change in the model before and after these cutoff points.

Next, we collect empirical evidence which establishes that the shape of the price path for routes with $\alpha_m = \alpha^L$ is u-shaped, if we discard the oldest five weeks of observations. We ran the following regression, where we introduced route-specific dummies and dummies for weeks as follows

$$P_{mt} = \delta_m + \beta_1 D^1 + \beta_2 D^2 + .. + \beta_7 D^7 + \varepsilon_{mt} \tag{19}$$

where, D^1 = dummy for the last two weeks before departure, D^2 = dummy for 3 to 4 weeks before departure, D^3 = dummy for 5 to 6 weeks before departure and so on. Results for this regression equation are displayed in table 6.

The coefficients for dummies D^1 to D^5 shows that prices fall and then rise as the date of departure draws closer. Thus, a u-shaped pattern emerges once we choose to concentrate only on the last 10 weeks before take-off. We ran similar regressions for routes with medium and high proportion of business travelers, and found evidence of a u-shape for routes with medium proportion of business travelers when we looked at the last 10 weeks before departure, while no such pattern emerged for routes with high proportion of business travelers where the price path was found to be rising.

	Coeff.	St. Error	t-stat	p-Value
D^1	65.679	6.352	10.34	0.000
D^2	29.089	5.810	5.01	0.000
D^3	28.918	6.344	4.56	0.000
D^4	42.968	6.718	6.40	0.000
D^5	45.904	6.522	7.04	0.000
D^6	19.993	6.765	2.96	0.003
D^7	−6.134	6.670	−0.92	0.358

Table 6: Regression Results for Equation 6 for Routes with $\alpha = \alpha^L$

8 Conclusion

We construct a three-period model in which a single seller facing a capacity constraint offers a finite measure of units of a non-durable good to a continuum of buyers, each of whom might be one of two possible types. The seller chooses without precommitment, prices and measure of units to offer for sale over the three periods in order to maximize discounted sum of revenue earned. We determine possible shapes of the corresponding price path for different values of the parameters and find that for certain combinations of the parameter values, the optimal price path is *u-shaped*. For other combinations, we find that the optimal price path is either *strictly non-decreasing* (which is consistent with a result in a paper by Stavins) or horizontal.

While the theoretical prediction that prices never fall before departure was corroborated, the prediction that the price path for routes with the highest proportion of "high" type buyers is horizontal was found to be empirically invalid. Instead, routes with high proportions of business travelers witnessed the steepest increase in prices. The price path for the routes with low and medium proportions of business travelers was also found to be increasing.

In our theoretical model we assumed that the proportion of buyers with a higher valuation for the good, who enters the market in each period, remains constant over the three periods. In reality, this is clearly not the case. It is our conjecture that a theoretical model which allows for variation in the proportion of high valuation buyers over the three periods, where the proportion increases from the first to the third period, will perform better in terms of providing an explanation for the empirical results. However, even if we do solve for the price paths for various parameter values for such a model, it will be difficult to access data which describe how the proportion of travelers traveling for business purposes on different routes change as the date of departure draws closer.

The theoretical model also predicted a small range of parameter values for which the price path would be u-shaped. While we did find some empirical evidence for a u-shaped price path for routes with low or medium proportion of high valuation buyers, we did so only after truncating the data and considering the last 10 weeks of observations.

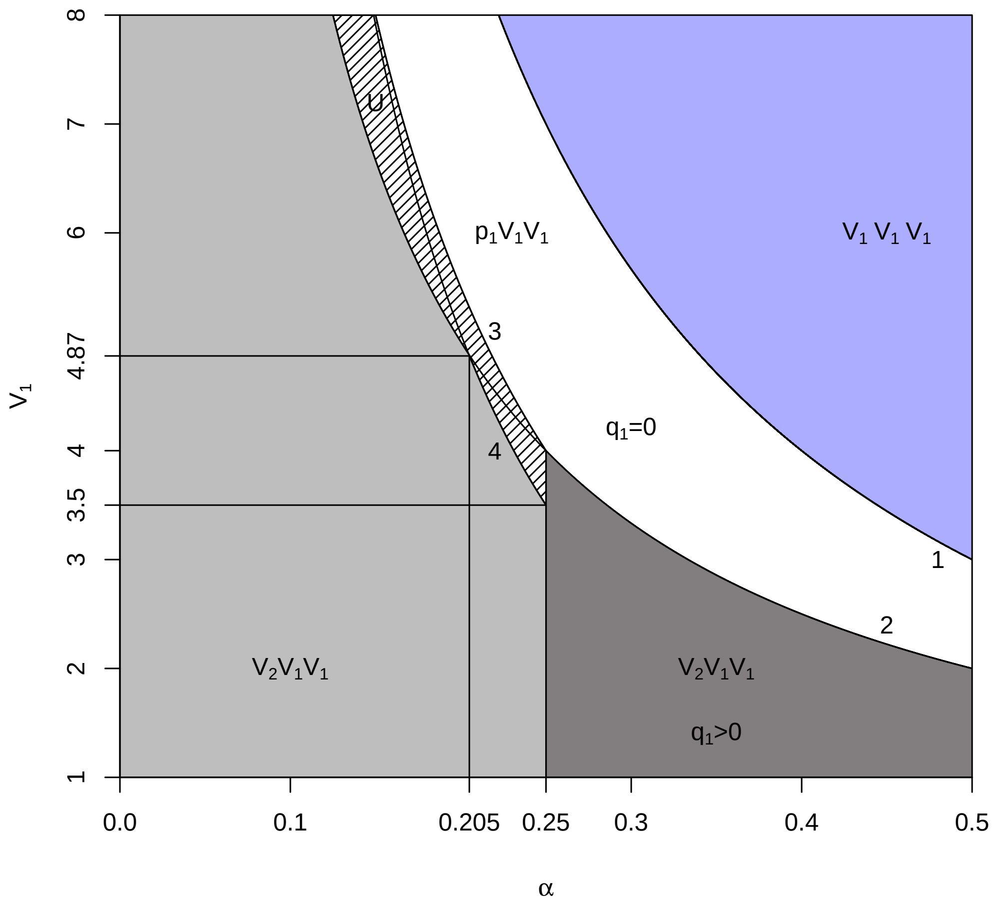

Figure 1: Subgame Perfect Outcomes for different values of V_1 and α ($V_2 = 1$).

(1) $V_1 = \dfrac{2-\alpha}{\alpha}$; (2) $V_1 = \dfrac{1}{\alpha}$; (3) $V_1 = \dfrac{3-4\alpha}{2\alpha}$; (4) $V_1 = \dfrac{6\alpha^2 - 7\alpha + 4}{\alpha(4\alpha+2)}$

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City of Origin	Destination City	Airline Carrier	Date of Departure
Detroit,MI	Orange County, CA	Northwest	June 6
Spokane, WA	Las Vegas, NV	America West	June 6
Austin, TX	Washington DC (IAD)[a]	United	June 6
Orlando, FL	Rochester, NY	Air Trans	June 6
Burbank, CA	Atlanta, GA	Delta	June 6
Detroit, MI	San Diego, CA	Northwest	June 6
Portland, OR	Santa Barbara, CA	Alaska	June 7
Wrangell, AK	Petersburg, AK	Alaska	June 7
Reno, NV	Orange County, CA	Aloha	June 7
Kansas City, MO	San Antonio, TX	Midwest	June 7
Akron, OH	Tampa, FL	Air Trans	June 7
Providence, RI	Fort Myers, FL	Spirit	June 7
Denver, CO	Little Rock, AK	Frontier	June 8
San Francisco, CA	Austin, TX	United	June 8
Santa Barbara, CA	Dallas, TX	American	June 8
Akron, OH	Orlando, FL	Air Trans	June 8
Cincinnati, OH	Orange County, CA	Delta	June 8
Birmingham, AL	Washington DC (DCA)[b]	Delta	June 8
Indianapolis, IN	Miami, FL	American	June 2
New Orleans, LA	Boston, MA	American	June 2
Pittsburgh, PA	Los Angeles, CA	US Airways	June 2
Cleveland, OH	San Antonio, TX	Continental	June 2
Seattle, WA	Tucson, AZ	Alaska	June 2
Miami, FL	Phoenix, AZ	America West	June 10
Memphis, TN	Las Vegas, NV	Northwest	June 10
San Francisco, CA	Kansas City, MO	Midwest	June 10
Dallas/Fort Worth	Providence, RI	American	June 10
Portland, ME	Charlotte, NC	US Airways	June 10
Phoenix, AZ *	Kahului, HI	ATA	June 10/17
Newark, NJ *	Honolulu, HI	Continental	June 10/17

Notes: (a) There were no direct flights to Ronald Reagan Washington National Airport (DCA) from Austin. (b) There were no direct flights to Dulles International Airport (IAD) from Birmingham. * Two-way flights, with June 17, 2005 as return date.

Table 7: Routes, Carriers and Dates of Departure

City of Origin	Destination City	Criteria (Misc.*)	Origin/Destn: CD/City
Detroit,MI	Orange County, CA	0.46	0.46
Spokane, WA	Las Vegas, NV	0.17	
Austin, TX	Washington DC (DCA)	0.7	0.62
Orlando, FL	Rochester, NY	0.03	0.03
Burbank, CA	Atlanta, GA	0.74	0.74
Detroit, MI	San Diego,CA	0.40	0.40
Portland, OR	Santa Barbara, CA	0.43	
Wrangell, AK	Petersburg, AK	0.29	
Reno, NV	Orange County, CA	0.15	0.11
Kansas City, MO	San Antonio, TX	0.52	0.41
Akron, OH	Tampa, FL	0.15	0.15
Providence, RI	Fort Myers, FL	0.17	
Denver, CO	Little Rock, AK	0.33	
San Francisco, CA	Austin, TX	0.41	
Santa Barbara, CA	Dallas, TX	0.55	0.52
Akron, OH	Orlando, FL	0.23	0.23
Cincinnati, OH	Orange County, CA	0.42	0.46
Birmingham, AL	Washington DC (DCA)	0.43	0.43
Indianapolis, IN	Miami, FL	0.06	0.45
New Orleans, LA	Boston, MA	0.55	0.55
Pittsburgh, PA	Los Angeles, CA	0.27	0.37
Cleveland, OH	San Antonio, TX	0.48	0.38
Seattle, WA	Tucson, AZ	0.25	
Miami, FL	Phoenix, AZ	0.10	0.35
Memphis, TN	Las Vegas, NV	0.15	
San Francisco, CA	Kansas City, MO	0.49	0.41
Dallas/Fort Worth	Providence, RI	0.29	
Portland, ME	Charlotte, NC	0.35	0.07
Phoenix, AZ	Kahului, HI	0.17	
Newark, NJ	Honolulu, HI	0.24	

Notes: * We use the proportion of travelers who traveled for business purposes (includes all forms of transportation) from or to the particular MA, with the CD or state or city as origin or destination.

Table 8: Proportion of “High” Type Buyers On Different Routes