Understanding fixed income instruments

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Goals

- What are fixed income securities?
- Pricing fixed income securities
- Interest rates – spot and forward rates

Definitions

Fixed income securities – definition

Any contract that defines a series of fixed cashflows coming at fixed times. These contracts do not have any state contingencies built into them – the cashflow is fixed.

- Types of securities: loans, bonds
- Types of users: firms, individuals, government

Jargon on Bond Street

A "coupon bearing bond" A bond that pays fixed cashflow every year, until it matures at date T when it also pays the **face value** of the bond.
Eg: B_1 with face value of Rs.100, maturity $T = 5$ and annual cashflow of Rs.10 looks like:
$(10, 1), (10, 2), (10, 3), (10, 4), (110, 5)$

A "zero coupon bond" A bond that pays Rs.100 at date T – it is a bond with no coupons.
Eg: A zero coupon bond, B_2 with face value of Rs.100 and maturity $T = 5$ looks like:
$(100, 5)$
This is also called "a pure discount bond".

Note: In the (cashflow, time point) pairs, time is expressed in years.

Bonds in India

- The largest bond issuer in India: Govt. of India (GOI).
- GOI bonds: semi-annual cashflows, maturities between 3months to 30 years.
- GOI bonds with maturity, $T <= 1$ year are called **Treasury bills.**
- Longer maturity bonds are called **Treasury bonds.**
- Other large issuers are state governments, municipalities, public sector companies, large private sector companies.
- Companies also issue bonds.
  These tend to have different cashflow structures:
  - Annual rather than semi-annual flows,
  - Coupons paid can be different every year rather than fixed,
  - At maturity, bond-holders may get the option to buy shares of the company rather than receive the face value of the bond.
  - Some corporate bonds give the issuing company the right to payback the cash early to the bond-holder.

Example of a GOI bond

Most traded GOI bond today: CG2020 7.80%. The features of the bond are as follows:

1. All GOI bonds have a face value of Rs.100.
2. Matures in 2020. Wrt today (ie, Jan 2010), it is a 10-year bond.
3. It pays 7.80% of the face value every year – ie, Rs.7.80 per bond owned.
4. The payments are made semi-annually. Every six months, the bond-holder gets Rs.3.90.
5. The last six-monthly payment coincides with the date of maturity of the bond. The last payment to the bond-holder becomes Rs.103.90.
6. The probability that GOI will default on the payment of the bond is assumed to be zero.

Cashflows of the CG2012 7.40 bond:
$((3.90, 0.5), (3.90, 1.0), (3.90, 1.5), \ldots, (103.90, 10))$

A bond, a portfolio: it's all cashflows

Case 1 I have two bonds:

1. $B_1 \rightarrow (10, 1), (10, 3), (100, 5)$
2. $B_2 \rightarrow (10, 2), (10, 4)$

Case II I have one bond:

$B_3 \rightarrow (10, 1), (10, 2), (10, 3), (10, 4), (100, 5)$

- There is no difference to the end cashflows to the investor.
- Take a portfolio, set union of all cashflows of all products, and you have one hypothetical bond.

Pricing

Present value

- What is a cashflow C, at date T worth today?

$$\text{NPV} = \frac{C}{(1+r)^T}$$

  Compound interest formula.
- With continuous compounding, we calculate NPV as

$$\text{NPV} = e^{-sT} C$$

  where we use $s = \log(1+r)$
- In either case, what we need is the rate r for T.

Pricing a general set of cashflows

Suppose we face a set of cashflows $(c_1, t_1), (c_2, t_2), \ldots, (c_N, t_N)$.

$$\mathrm{NPV} = \sum \frac{c_i}{(1 + z(t_i))^{t_i}}$$

Concept of term structure

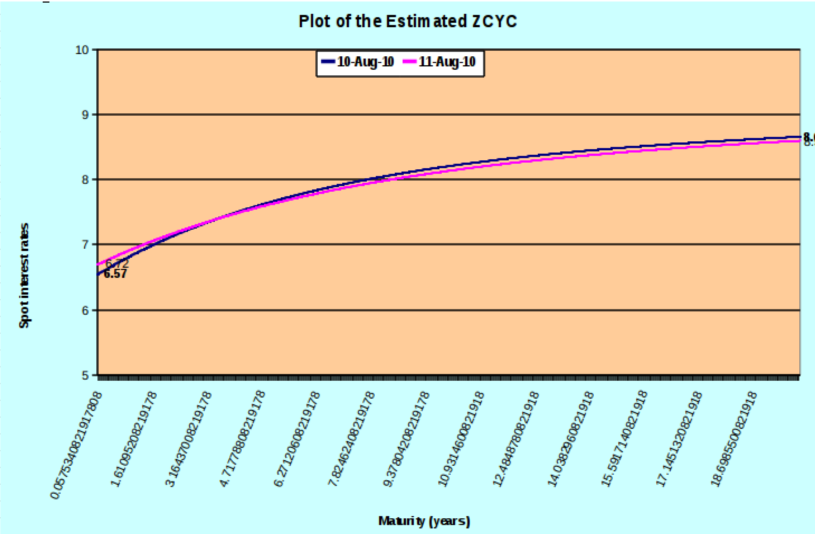
- Interest rate for a cashflow at t is different from what is used for a cashflow at $(t + 1)$.
  Eg, Rs.100 to be received one year later has a different value today compared to the same Rs.100 to be received ten years later.
- In that case, the NPV notation used for a cashflow C at date T is:
  $$\text{NPV} = \frac{C}{(1 + z_t)^T}$$
- This incorporate the idea of varying interest rates by maturity.

Interest rates

Not one interest rate, but a whole term structure

- One unique "interest rate in the economy" does not exist.
- There is a different interest rate for every different maturities, $z(t)$.
- The graph of maturity t on the x-axis and corresponding interest rates z_t on the y-axis is called the **term structure of interest rates**.
- In general, this graph is not flat.
- It is also called the **spot yield curve** or the **zero coupon yield curve (ZCYC)**
- Every day, we get a different yield curve $z(t)$.
- The coupon rate used in recent government bond issuance has nothing to do with interest rates.

Term structure of interest rates for India 18^{th}, 19^{st} January 2010



The spot yield curve $z(t)$ is the core pricing technology

Once we know the ZCYC $z(t)$, we can price every cashflow.

- Zero coupon bonds are easy.
- Coupon bearing bonds are straightforward.
- Linearity in pricing bonds holds.
  Suppose we face two bonds:
  1. $(10, 1), (10, 3), (100, 5)$ with NPV p_1
  2. $(10, 2), (10, 4)$ with NPV p_2

  Then the whole "bond portfolio":
  $(10, 1), (10, 2), (10, 3), (10, 4), (100, 5)$ has NPV $p_1 + p_2$.
  This is a very simple and attractive property of a pricing technology.
  The ZCYC has this property, some other methodologies do not.

Estimation of the ZCYC

- Interest rates are discovered from market prices of traded bonds.
- Suppose we have a set of bonds in the country. Each bond is a known list of cashflows and timepoints.
- The secondary market reveals a price for each bond.
- However, each bond captures several interest rates all captured by one price.
  How do we discover the term structure of interest rates from one price?
  1. Start with choosing a functional form $z(t;\theta)$ for the zcyc.
  2. We need to estimate the parameters θ which give us a "best fit" for the data:

$$\min_{\hat{\theta}}(\text{price}_{\text{model}} - \text{price}_{\text{market}})^2$$

  3. Estimation by **MLE**

Choices of functions for the term structure of interest rates

- There are multiple approaches to estimating the ZCYC:
  - **Empirical**: polynomial splines (McCullogh 1971), exponential splines, B-splines (Vasicek and Fong 1982, Eom, Subrahmanyam and Uno, 1998), Nelson-Siegel function (Nelson and Siegel 1987, Svensson 1995), Fama-Bliss function (Fama and Bliss 1997).
    Requires market information of bond prices, across a spectrum of maturities.
  - **Theoretical**: general equilibrium models (Vasicek 1977, Cox, Ingersoll and Ross 1985, Longstaff and Schwartz 1992) or no–arbitrage models (Ho and Lee 1986, Heath, Jarrow and Morton 1990, Hull and White 1990).
    Requires market information on the level and the volatility of the short-term interest rate and forward rates.
- In India, paucity of information about market interest rates make us choose a parsimonious, empirical model – the Nelson Siegel.

Data sources on ZCYC for India

- In India, NSE-WDM and CCIL calculate a daily ZCYC from market prices of GOI bonds.
- NSE-WDM is the *Wholesale Debt Market* at the *National Stock Exchange*.
- CCIL is the *Clearing Corporation of India Ltd.*
- Time series of daily ZCYC from 1/1/1997 onwards.

ZCYC vs. YTM

Competing concept of YTM or IRR

- Suppose there is a bond with cashflows $(c_1, t_1), \ldots (c_N, t_N)$.
- And *suppose we observe a market price* $\bar{p}$ for the bond.
- We can ask: Suppose the yield curve was flat $z(t) = a_0$. What should a_0 be so that

$$\bar{p} = \sum \frac{c_i}{(1 + a_0)^{t_i}}$$

- Fact: For every price $\bar{p}$, there exists an interest rate a_0 s.t. this equation is exact.
- Compute $\bar{a}_0$ by bisection.
- This interest rate is called “ytm” (Yield To Maturity) or “irr” (Internal Rate of Return).
- A historical curiosity with little role in finance.

Forward Interest Rates

Concept of forward rates

- Interest rate on a loan that starts at a future date t_1 and runs till future date t_2.
- This is written $r_{t_1}^{t_2}$.
- If a loan starts today, then $t_1 = 0$, and we have r_0^t which are "spot rates". This is why the zero coupon yield curve is called "the spot yield curve".

Computing forward rates

Case 1 I give you a loan (C1) for a two-year period at an interest rate r_0^2.

Case 2 I give you a loan (C2) for a one-year period at an interest rate r_0^1, *and*

we contract that at the end of the year, there will be a second loan (C3) for a one-year period at an interest rate r_1^2.

- Note that the contract is perfectly binding including actions at future dates.
- The two cases are identical. So we must have

$$\begin{aligned} \text{NPV of cashflow 1} &= \text{NPV of sum of cashflow2, cashflow3} \\ (1 + r_0^2)^2 &= (1 + r_0^1)(1 + r_1^2) \end{aligned}$$

- To know the forward rate r_1^2 I just need two spot rates r_0^1 and r_0^2! *The ZCYC embeds all possible forward rates.*